SPIN PRECESSION IN A 2-BODY SYSTEM: A NEW TEST OF GENERAL RELATIVITY

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1. Newtonian Theory
2. General Relativistic Corrections
    a. Periastron Precession
    b. Spin Precession
3. Relativistic Spin Precession in the Double Pulsar [Breton et al., *Science* 321, 104 (2008)]
4. Double Pulsar – Numbers

   (Spin Precession ~5°/yr. etc.)
5. Chart Comparing Various 1 & 2 Body Systems
6. GPB and Other Tests
7. Theoretical Approach - Outline

NEWTONIAN (1687/KEPLER (1619)

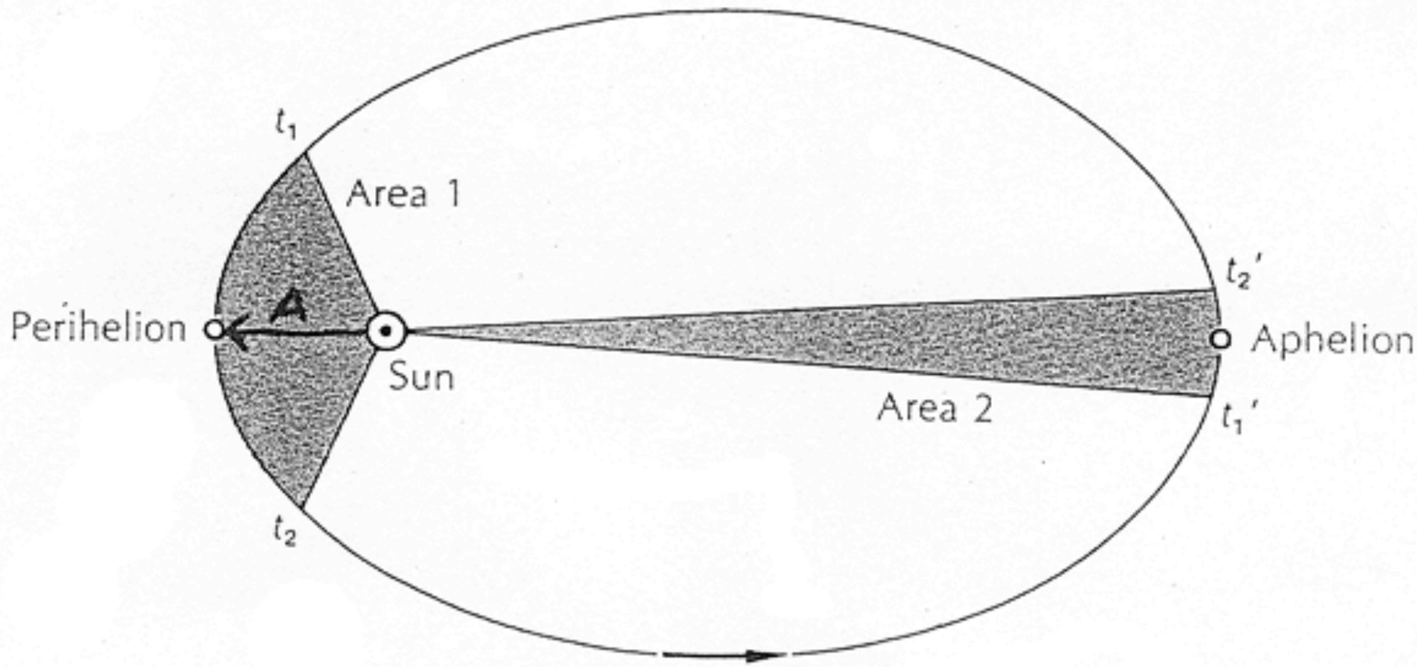


FIGURE 5-12. Kepler's law of equal areas: the two shaded areas are equal, so the speed of the planet is greater near perihelion because it must sweep out a larger arc of the ellipse in the time $t_2 - t_1$ compared to the equal time $t_2' - t_1'$.

$$F = \frac{Gm_1 m_2}{r^2} \Rightarrow \text{Closed Orbit (Eclipse)}$$

$\vec{L}$ = Orbital Angular Momentum $\rightarrow \perp$ to orbit

$\vec{\mathrm{A}}$ = Runge - Lenz Vector

$\rightarrow$ From focus to perihelion

$\rightarrow \perp$ to $\vec{L}$

$$\boxed{\frac{d\vec{L}}{dt} = 0} \qquad \boxed{\frac{d\vec{A}}{dt} = 0}$$

GENERAL RELATIVITY: 1-BODY/NO SPIN

Einstein field equations

$\downarrow$

Schwarzschild solution

$\downarrow$ Post-Newtonian Approx.

$\downarrow$ (one order beyond Newton)

$\downarrow$

$$\boxed{V = -\frac{Gm_1m_2}{r}\left\{1+\frac{6Gm_2}{c^2r}\right\}}$$

$\longrightarrow$ Perihelion Precession (of $\vec{A}$)

DEF: $\alpha_G \equiv \frac{Gm_2}{c^2 a} \equiv \frac{r_s}{a}$

$=$ Gravitational Fine – Structure Constant

(Analogous to $\alpha = \frac{e^2}{\hbar c}$ in QED)

$$\boxed{\frac{d\vec{A}}{dt} = \vec{\Omega}^{*(E)} \times \vec{A}} \qquad \boxed{\frac{d\vec{L}}{dt} = 0}$$

Average Rotational Angular Velocity

$$\boxed{\vec{\Omega}^{*(E)} = \left(\frac{3}{1-e^2}\right)\alpha_G \overline{\omega} \hat{L} \qquad \sim \alpha_G \overline{\omega}}$$

$$\xrightarrow{MERCURY-SUN} \approx 3\times\left(3\times10^{-8}\right)\left(8.3\times10^{-7}\right) rads/s = \boxed{\text{43''/CENTURY}}$$

2. GENERAL RELATIVISTIC CORRECTIONS

DEF: $a = \text{Semi} - \text{Major Axis}; \;\; e = \text{Eccentricity}$

$$M = m_1 + m_2$$

$$\mu = \frac{m_1 m_2}{M}$$

$$\vec{L} = L\vec{n} \, (\vec{n} = \hat{L}) = \text{Angular Momentum}$$

$$\vec{A} = \text{Runge} - \text{Lenz Vector}$$

$$\boxed{\alpha_g} = \frac{GM}{c^2 a} = \text{Gravitational Coupling Constant} = \frac{(\overline{\omega} a)^2}{c^2} = \left(\frac{v_{av}^2}{c^2}\right)$$

$$\boxed{\overline{\omega}} = \frac{2\pi}{T} = \left(\frac{GM}{a^3}\right)^{1/2} = \frac{L/\mu}{a^2\left(1-e^2\right)^{1/2}} = \text{Average Orbital Angular Velocity}$$

(T = Period)

$$\boxed{\textbf{ALL}\ \text{precessions per unit time} \quad \sim \alpha_g \overline{\omega}}$$

(a) Periastron precession (Robertson/EIH)

$$\frac{d\vec{A}}{dt} = \left(\vec{\Omega}^{*(E)} \times \vec{A}\right) = \left(\frac{d\omega}{dt}\right)\left(\vec{n} \times \vec{A}\right)[\omega = \text{Argument of Perihelion}]$$

$$\boxed{\frac{d\omega}{dt} = \frac{3}{1-e^2}\left(\alpha_g \overline{\omega}\right)}$$

(b) Spin Precession (Barker-O'Connell)

$$\frac{d\vec{S}^{(1)}}{dt} = \left(\vec{\Omega}_{so}^{(1)} \times \vec{S}^{(1)}\right) + \text{Spin} - \text{Spin term (smaller)}$$

$$= \left|\vec{\Omega}_{so}^{(1)}\right| \left(\vec{n} \times \vec{S}^{(1)}\right)$$

$$\boxed{\begin{aligned} \left|\vec{\Omega}_{so}^{(1)}\right| &= \frac{3}{2\left(1-e^2\right)} \frac{\left(m_2 + \frac{\mu}{3}\right)}{M} \left(\alpha_g \overline{\omega}\right) \\ &= \left[\frac{1}{2} \frac{\left(m_2 + \frac{\mu}{3}\right)}{M}\right] \frac{d\omega}{dt} \end{aligned}}$$

$$\xrightarrow{IF \;\; m_2 >> m_1} \frac{1}{2} \frac{d\omega}{dt}$$

GENERAL CHECKS

1. $$\frac{d\vec{A}}{dt} = \vec{\Omega}^* \times \vec{A}$$

$$\frac{d\vec{L}}{dt} = \vec{\Omega}^* \times \vec{L}$$

Hence

$$\frac{d}{dt}\left(\vec{A} \cdot \vec{L}\right) = 0$$

$\Rightarrow \vec{L}$ is always $\perp$ to $\vec{A}$, as is required

GENERAL CHECKS

2. $$\frac{d\vec{J}}{dt} = \frac{d}{dt}\left\{\vec{L} + \vec{S}^{(1)} + \vec{S}^{(2)}\right\} = 0$$

Conservation of total momentum

Science **321**, 104 (July 4, 1008)

Relativistic Spin Precession in the Double Pulsar

Breton et al.,

<u>measure the relativistic precession of pulsar B's spin axis</u>

4.77°/yr observed

5.07°/yr theory [4°.78 for *A*]

↓ ↓

PERIODS → **<u>71 yrs</u>** **<u>75 yrs</u>**

B. M. Barker and R. F. O’Connell, *Phys. Rev. D* **12**, 329 (1975)

↓

“predicted by general relativity”

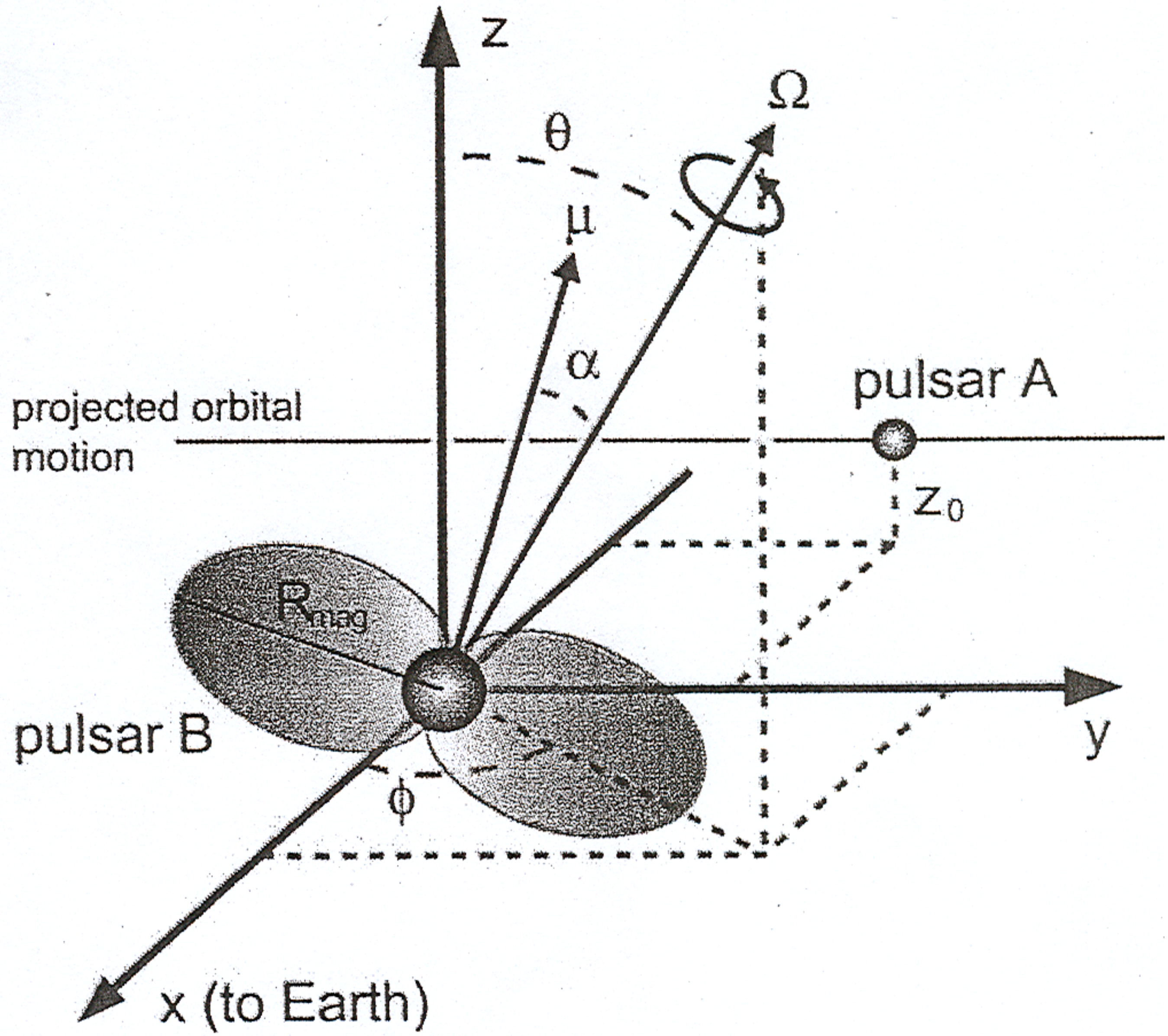
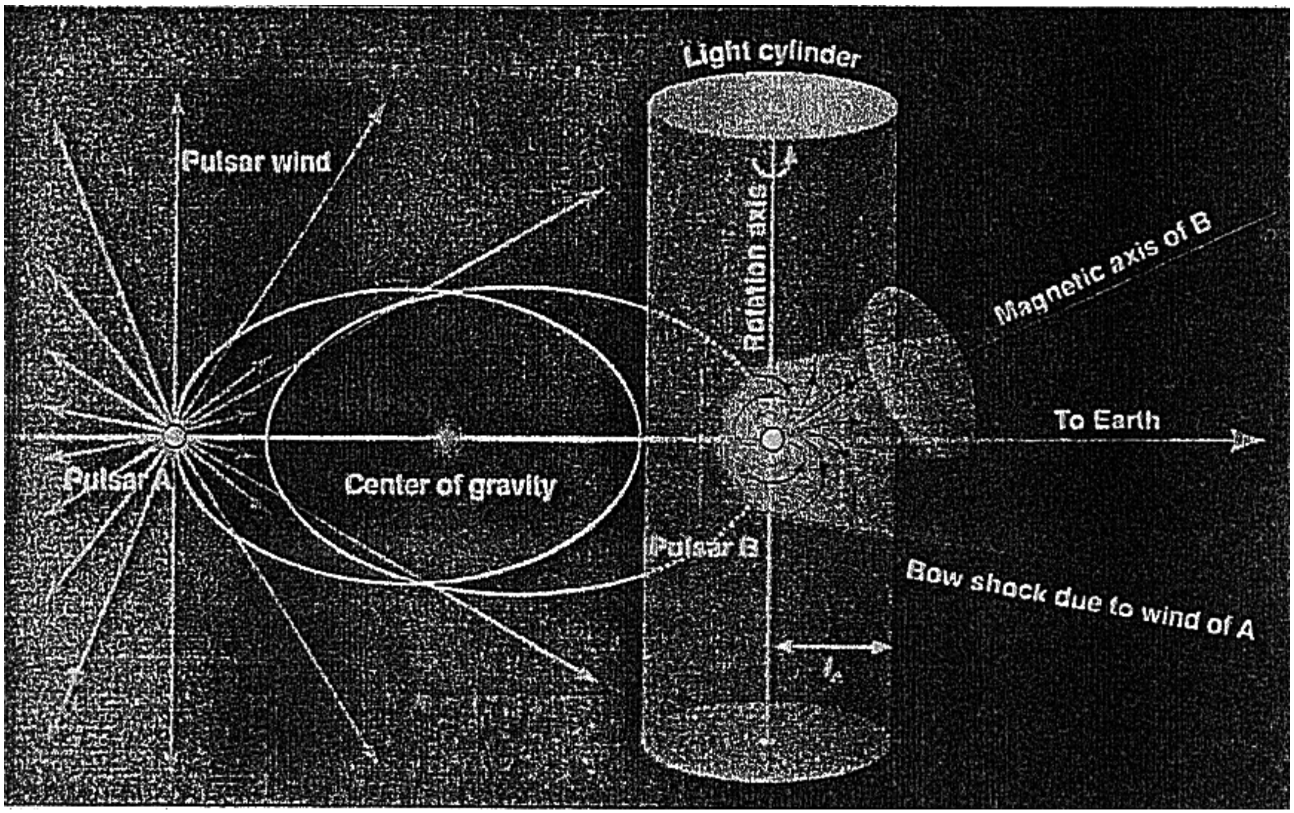

z
Ω
θ
μ
α
pulsar A
projected orbital motion
z_0
R_{mag}
pulsar B
y
φ
x (to Earth)

Figure 1: Schematic view of the double pulsar system showing the important parameters for the modeling of pulsar A's eclipse (dimensions and angles are not to scale). Pulsar B is located at the origin of the cartsian coordinate system, whereas the projected orbital motion of pulsar A during its eclipse is parallel to the y axis at a constant z_0 as seen from Earth, which is located toward the positive x axis. Because the orbital inclination is almost perfectly edge-on *(14)*, we can approximate the z axis to be coincident with the orbital angular momentum. The spin axis of pulsar B, whose spatial orientation is described by θ and ϕ, is represented by the Ω vector. The magnetic axis of pulsar B corresponds to the μ vector and makes an angle with respect to Ω. Lastly, the absorbing region of the diplar magnetosphere of pulsar B, truncated at radius R_{mag}, is shown as a shaded red region.

10. DOUBLE PULSAR SYSTEM: THE WORKS

Science 303, 1143 (Feb. 20, 2004)



A is ≈ 3,600 times as energetic as B!!

Two-Pulsar dance. Schematic of the double pulsar system (not to scale) relative to observers on Earth. The ellipses are the orbits of the two pulsars A and B around the common center of gravity seen at an oblique angle.

Pulsar A's strong outflow of relativistic particles and magnetic fields ("pulsar wind")penetrates into the light cylinder of star B and causes formation of a bow shock with long tail behind pulsar B. The light cylinder (with radius l_c) plays an essential role in the generation of the radio beams that cause the observed pulsed signal.

The beam of pulsar B is depicted here as a hollow cone centered on the magnetic pipole axis. The distruption of pulsar B's light cylinder on the side facing pulsar A may short-circuit the currents in B's magnetosphere that produce the radio beams, which might explain the weakness of the pulses of B observed over most of its orbital cycle.

Changes in orientation of the light cylinder will cause <u>variations in the emitted beam</u>, as will <u>relativistic precession of the rotation axis</u>.

$l_c = \frac{c}{\Omega^{spin}} \rightarrow$ co-rotation of charged particles in magnetosphere cannot persist beyond the surface where the tangential velocity = c

4. DOUBLE BINARY NUMBERS

Distance to Earth 2.2×10^{16} miles

$\approx \left(\frac{1}{22}\right)$ Milky Way Diameter

$$\underline{R_\Theta \approx 7 \times 10^2\, km}$$

$$n - \text{Star Radius} = 10\, km$$

$$\approx 10^{-5} R_\Theta$$

Number of Pulsars $\approx 1{,}700$

$\rightarrow$ only 3 in nearby galaxies

Binary with 1 Pulsar and 1 Neutron star 6

Double Pulsar 1 (Very Unique!)

$$a = \text{Semi - Major Axis} = 1.25 R_\Theta = 8.7 \times 10^5\, km$$

$\boxed{\Rightarrow \text{Entire Binary could fit within our Sun}}$

$$e = \text{Eccentricity} = 0.088$$

$$i = \text{Angle of Inclination} = 88.69^{\circ}$$

$$\boxed{\Rightarrow \text{System is observed nearly perfectly edge-on}}$$

$$T = \text{Orbital Period} = 2.45 \text{ Hours}$$

Pulsar	**Mass**	**Pulse Period**	**Pulse Frequency**	**Surface B field (G)**	**Magneto-sphere Extent**	****** **Spin Precession**	**Time for 360º Revolution of Spin**
$A(m_2)$	1.3381	22.7	$44 s^{-1}$	6×10^{9}	$1{,}084\,km$	$4^{\circ}.782$	75 years
$B(m_1)$	1.2489	2773.5	$0.36 s^{-1}$	2×10^{12}	$132{,}000\,km$	$5^{\circ}.0734$	71 years

$$\boxed{\text{Periastron Precession}\left(\frac{d\omega}{dt}\right) = 16.89949(68)/\text{Year} \approx (43{,}000) \text{ PP of } Hg}$$

$$\approx m_1 + m_2 = 2.58708(16) \rightarrow \text{High Accuracy}$$

$\boxed{\text{Spin Precession}} \rightarrow$ need separate values for m_1 and m_2

$\rightarrow \dfrac{m_1}{m_2}$ is obtainable from $a_1 = \dfrac{m_2}{M} a$ and $a_2 = \dfrac{m_1}{M} a$

and the measured projected semi-major values $\left(\dfrac{a_1 \sin i}{c}\right)$ and $\left(\dfrac{a_2 \sin i}{c}\right)$

$\rightarrow m_1$ and m_2 from **Shapiro Time Delay** [delay of $6.2\mu s$ due to A]

$\rightarrow$ **13% Accuracy** with improvements very likely

EMISSION

$B \approx 10^8 G - 10^{10} G$

$\Rightarrow$ Radio Emission

BIRTH

Supernova Explosions of Stars

$\approx (16\text{-}18) M_\Theta$ [$(4-10) M_\Theta$ for solitary pulsars]

TABLE COMPARING VARIOUS SYSTEMS

$$\alpha_g = \frac{GM}{c^2 a} \text{ where } M = m_1 + m_2; \; a = \text{Semi Major Axis}$$

$$\overline{\omega} = \frac{2\pi}{\mathrm{T}} = \left(\frac{GM}{a^3}\right)^{1/2} \text{ where } T = \text{ Orbital Period}$$

$$1^{\mathrm{o}}/\,\mathrm{yr} = \left(3\cdot 6\times 10^2\right) \text{ arc - sec/yr}$$

$$1 \text{ rad/sec} = \left(1\cdot 8\times 10^9\right)^0 \text{/yr} = \left(6\cdot 5\times 10^{12}\right) \text{ arc - sec/yr}$$

System	Sun-Mercury	Earth-Gyro (GP-B)	PSR 1913+16	* PSR * J0737-3039A/B (Double Binary)	Sun-(Earth/Moon) Gyro
α_g	3×10^{-8}	7×10^{-10}	2×10^{-6}	4.4×10^{-6}	9.8×10^{-9}
$\varpi\left(s^{-1}\right)$	8.3×10^{-7}	10^{-3}	2.2×10^{-4}	$7\times10^{-4}\,s^{-1}$	2×10^{-7}
$\alpha_g\varpi\left(s^{-1}\right)$	2.5×10^{-14}	7.5×10^{-13}	4.4×10^{-10}	$3.1\times10^{-9}\,s^{-1}$	1.9×10^{-15}
$\boxed{\alpha_g\varpi\left({}^\circ/\text{yr}\right)}$	$\left(4.5\times10^{-5}\right)^\circ/\text{yr}$	$\left(1.4\times10^{-3}\right)^\circ/\text{yr}$	$(0.79)^\circ/\text{yr}$	$(5.6)^\circ/\text{yr}$	$12\,mas/\text{yr}$
Periastron Precession (P.P.) $\dfrac{3\alpha_g\varpi}{1-e^2}$	43"/century	$(12.6)''/\text{yr}$	$4^\circ.2/\text{yr}$	$16^\circ.9/\text{yr}$	$36\,mas/\text{yr}$
Spin-Orbit Precession $\approx\frac{3}{2}\left(\alpha_g\varpi\right)\approx\frac{1}{2}$ (P.P.)	21"/century [irrelevant due to strong tidal interaction with sun]	$6.6''/\text{yr}$ [Spin-Spin is $0.04''/\text{yr}$]	$1^\circ.1/\text{yr}$	$5^\circ.1/\text{yr}$ (Pulsar B) $4^\circ.8/\text{yr}$ (Pulsar A	$19\,mas/\text{yr}$
a	$6\times10^{7}\,kms$	$6\times10^{3}\,kms$	$2\times10^{6}\,kms$	$9\times10^{5}\,kms$ $= 1.25\;\; R_\Theta$	$1.5\times10^{8}\,km$
M/M_Θ	1	3×10^{-6}	1.44 (Pulsar) 1.39 (Companion)	$1.34(\text{A})\rightarrow m_2$ $1.25(\text{B})\rightarrow m_1$ $M_\text{A}/M_\text{B}=1.07$	1
e	0.206	0.0014	0.6	0.08	0.017

OBSERVATIONS $\left[1mas = \left(2\cdot 8\times 10^{-7}\right)^{o}\right]$

i) <u>Single Pulsar</u> $\rightarrow$ Stairs, et al., *PRL* <u>93</u>, 141101 ('04)/ $0\cdot 51^{o}$/yr (95% confidence)/2-Body

ii) <u>Lense-Thirring effect on 2 lageos satellites [spin (earth)-earth orbit]</u>
Sattellite-Laser-Ranging (precession $\approx$ few mms.) $\rightarrow$ Ciufolini and Paulis, *Nature* <u>431</u>, 958 ('04)/ 31 (and $31\cdot 5$) mas/yr / Nodal (orbital) Motion of satellite

iii) <u>Lunar Laser Ranging [Spin (earth-moon)-Sun orbit]</u>
$\rightarrow$ JPL etc., *PRL* <u>98</u>, 071102 ('07)
$\rightarrow$ 0.1% Accuracy/Nodal motion of moon

iv) <u>Double Pulsar</u> $\rightarrow$ Breton et al., Science 321, 104 (2008)
$\rightarrow$ 5.07%/yr (13% precision) [Agrees with B-O'C, *PRD* <u>12</u>, 329 ('75)]

v) <u>Gravity Probe B</u> $\rightarrow$ gyro orbiting earth
$\rightarrow$ NASA Final Report, Dec. 2008
Spin-Orbit (Geodesic) $\rightarrow$ $6{,}614\cdot 4\,mas$/year (error $0\cdot 5\%$)
Spin-Spin (Frame Dragging) $\rightarrow$ 39*mas*/year (error 15%) [170 times smaller]