Satellite Laser Ranging and Relativity

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First International LARES Workshop
Sapienza, Universitá di Roma
July 3-4, 2009

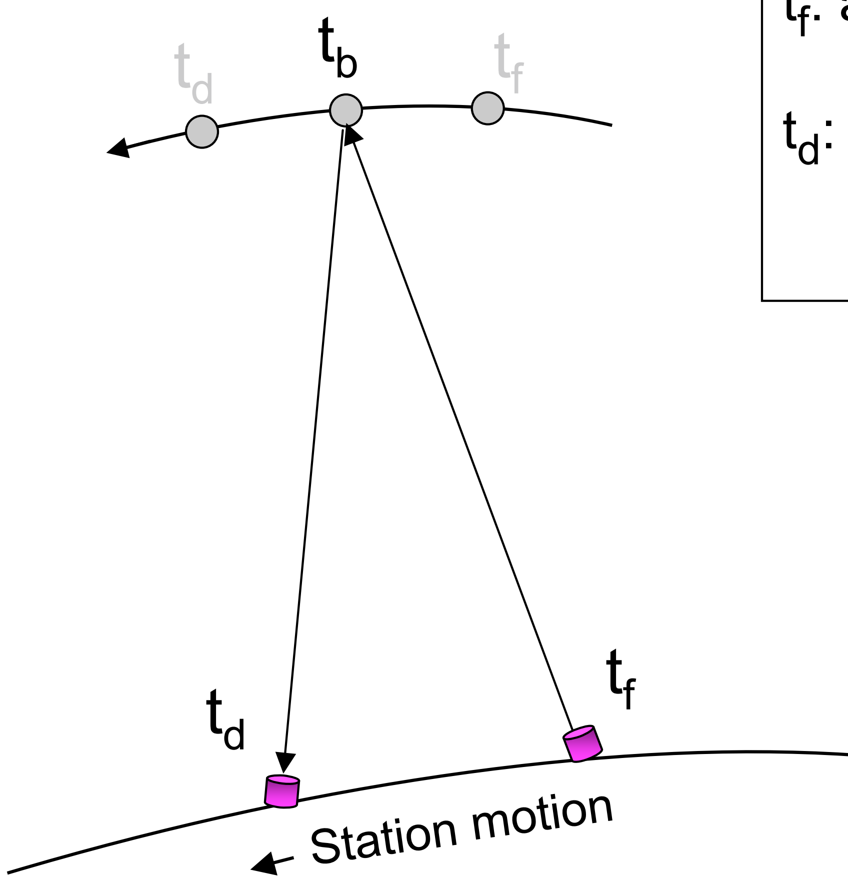
Overview

- With a measurement accuracy for satellite laser ranging (SLR) better than the part-per-billion level, the effects of General Relativity must be considered.
  - These include additional perturbations to the orbit dynamics, corrections to the light-time computation, and fundamental aspects of the definition of the geocentric reference frame.
- While these effects are significant, they are generally not large enough to provide tests of General Relativity competitive with those available from Lunar Laser Ranging and other solar system tests.
  - An important exception, however, is the relativistic prediction of the Lense-Thirring orbit precession, i.e the effect of 'frame-dragging' on the satellite orbit due to the spinning Earth's mass.

Satellite Laser Ranging

t_f: an ultra-short laser pulse is fired

t_d: laser pulse (as little as a single photon) is detected and the interval (t_d-t_f) is measured to 10-100 picosec accuracy

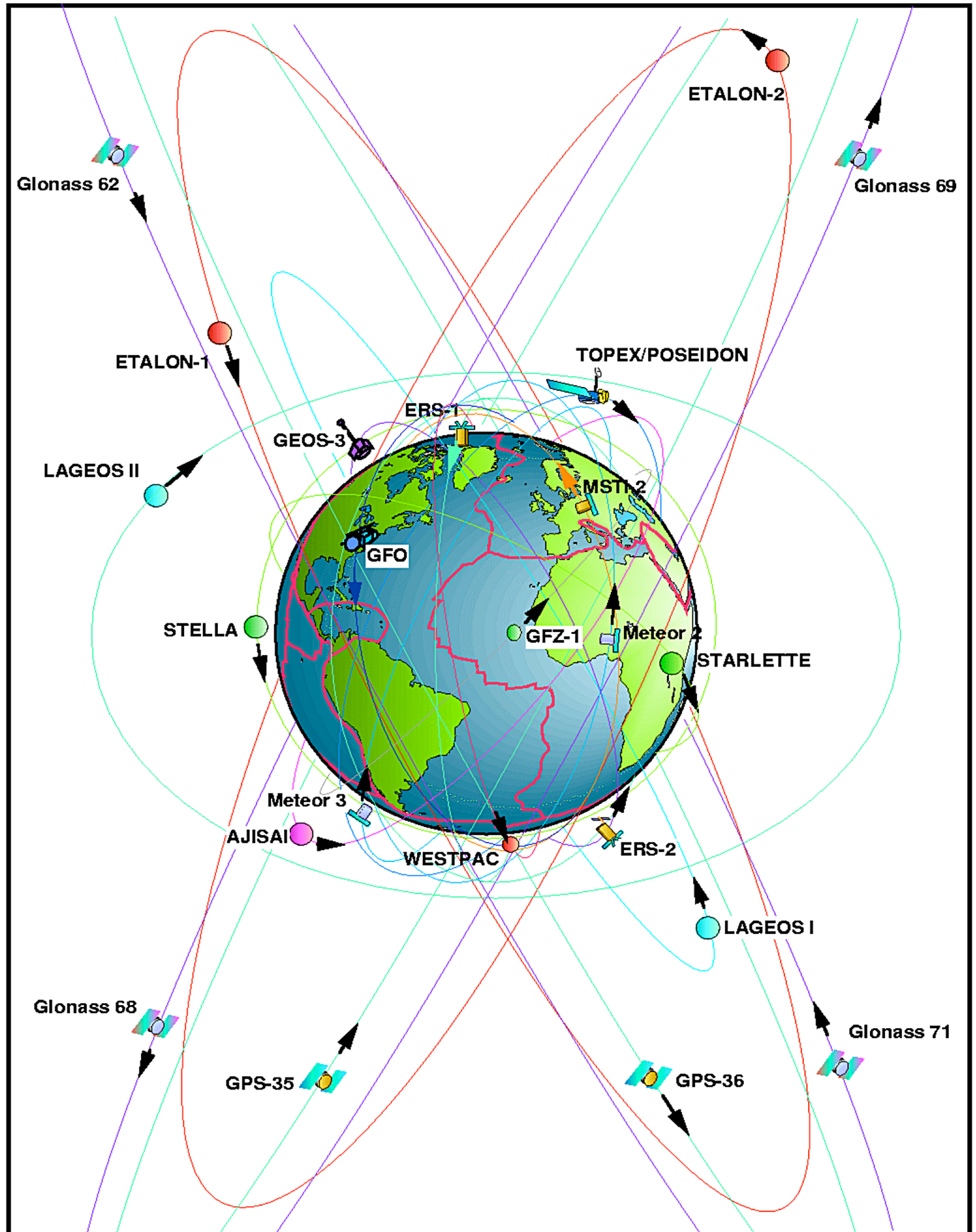


total round-trip light-time may be just a few milliseconds (or more than 2 seconds for lunar laser ranging)

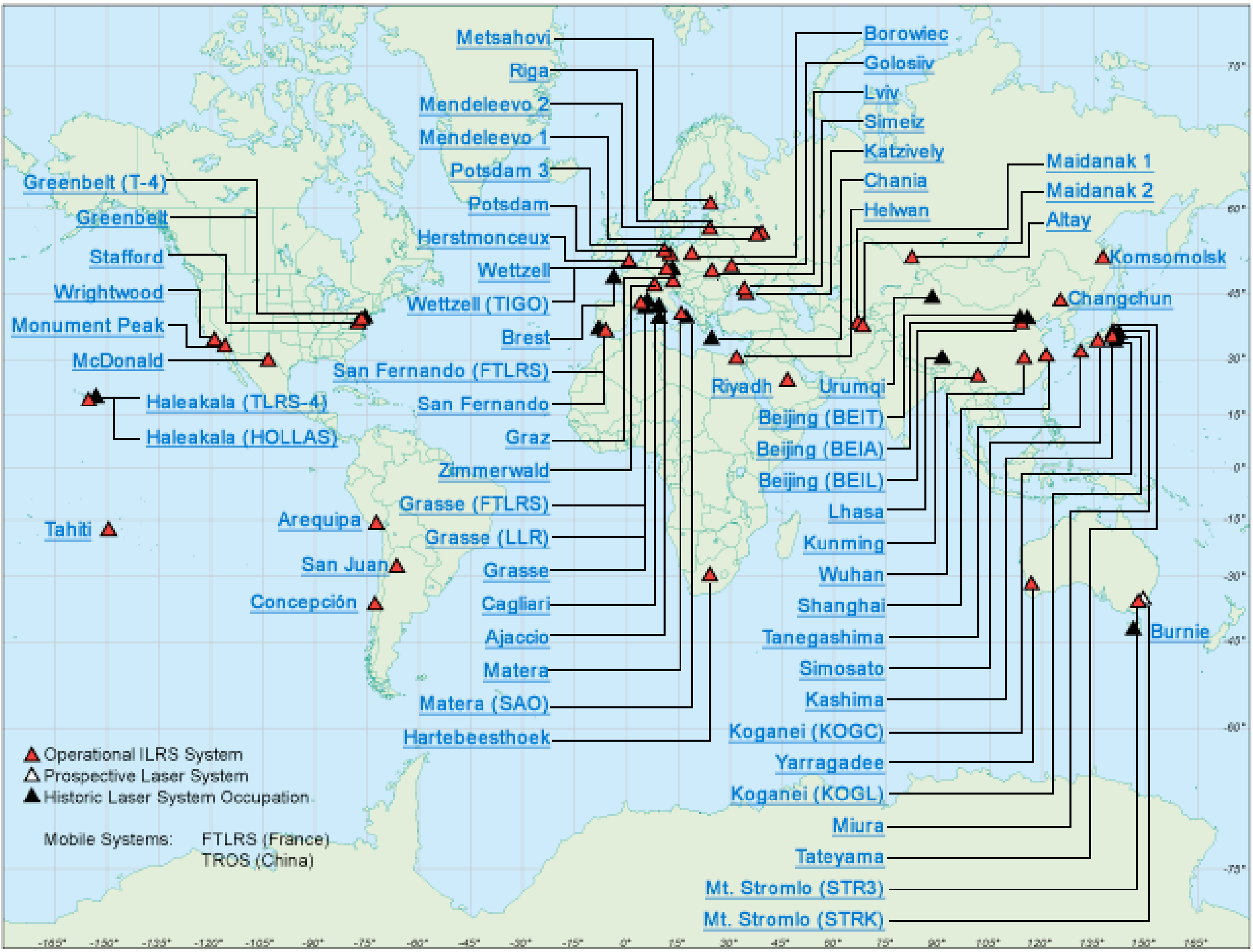
Laser tracking at Monument Peak, CA

Constellation of laser ranging satellites

LAGEOS and LAGEOS-2 are 60-cm diameter spheres with 426 corner cubes and ~400 kg mass



International Laser Ranging Service (ILRS)

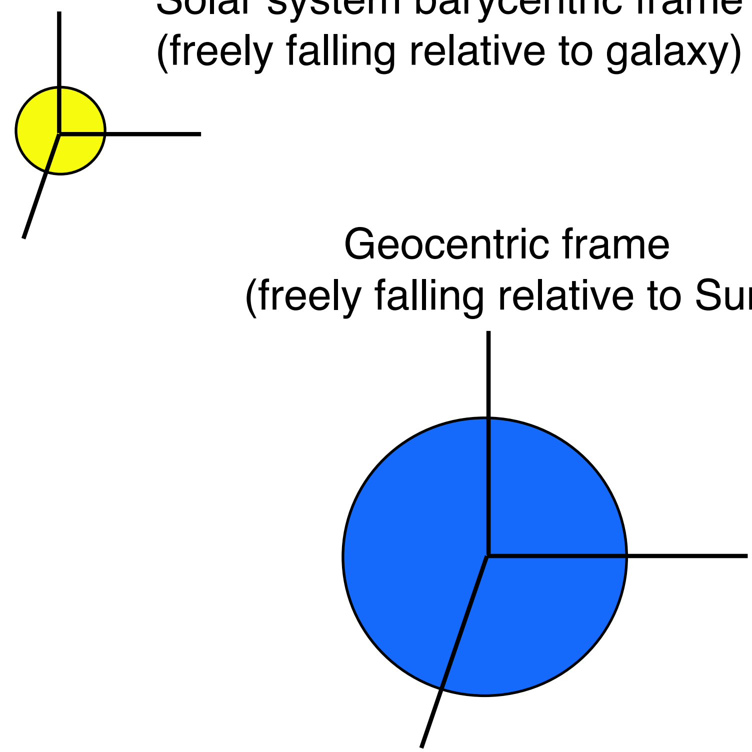


Relativity Considerations

- A ‘laser range’ is simply a measurement of an interval of proper time between two events (scaled by the speed of light)
- The model for that observation will depend on reference frame definition
  - Clocks and coordinate time
  - Dynamical effects
  - Observational effects

Solar system barycentric frame
(freely falling relative to galaxy)

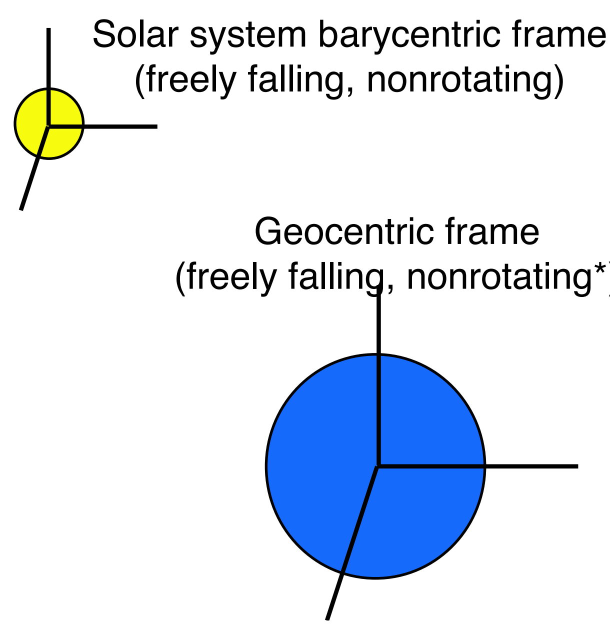
Geocentric frame
(freely falling relative to Sun



Within a region sufficiently close to their origins, both frames are valid inertial frames

Reference Frame Choice

- Calculations of physically observable effects will agree in the two reference frames if sufficiently close to geocentric origin and the model is complete
- Due to the high degree of cancellation of the relativistic effects of the Sun, geocentric model is sufficient for Earth satellites ➡ simpler model
  - The further from the Earth center, the poorer the cancellation
  - Geocentric frame would not be adequate for Lunar Laser Ranging
- Remaining relativistic effects due to Earth
  - Time delay due to Earth’s mass
  - Precession of perigee
  - Lense-Thirring precession
  - Geodesic precession*



Relativity Models for SLR

- Solar-system Barycentric model
  - Time transformation (periodic terms up to 1.6 msec)
  - N-body relativistic perturbing acceleration + relativistic geopotential correction
  - Light time correction due to Sun+Earth+Moon+…
  - Barycentric light time solution (accounts for motion of satellite and Earth in solar system during transit time)
  - Station position transformation (‘relativistic tides’)
  - Lense-Thirring precession
- Geocentric model
  - No time or spatial transformations
  - Schwarschild (1-body) relativistic perturbation
  - Light time correction due only to Earth
  - Geocentric light time solution (accounts only for motion of satellite and station motion during transit time)
  - Lense-Thirring and geodesic precession

Time Systems (1)

- TAI (International Atomic Time) is actually a 'paper' clock determined by averaging an ensemble of atomic clocks. The offset between each clock and TAI is monitored by broadcast time transfer.
- UTC (Coordinated Universal Time) is typically the time system to record the epochs of our observations. UTC is obtained from TAI by adding an integer number of seconds and is kept close to UT1.
  - Including the time zones differences results in civil time (wall clock time).
- TDT = TAI + 32.184 seconds is the practical realization of Terrestrial Dynamical Time (TDT).
- TDB is transformed to Solar-system Barycentric Dynamical Time (TDB) using only periodic variations.

UTC
leap seconds
+
↑
TAI
+
32.184 sec
↓
TDT
+
periodic terms only
↓
TDB

Time Systems (2)

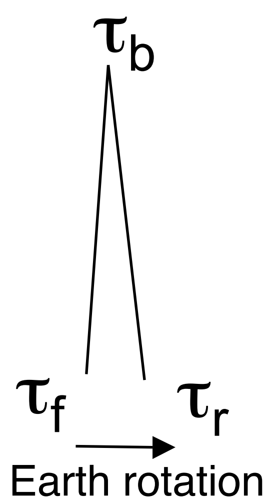
- In 1991, the time systems were redefined to explicitly include rate terms between terrestrial time (TT), geocentric coordinate time (TCG) and solar system coordinate time (TCB).

Rates:	between TT and TCG	$\sim 0.7 \times 10^{-9}$
	between TCG and TCB	$\sim 14.8 \times 10^{-9}$
	between TT and TCB	$\sim 15.5 \times 10^{-9}$

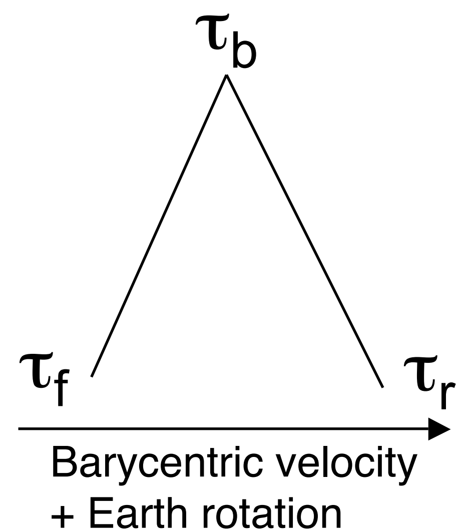
- Implementation of these new definitions has been impractical (particularly with the widespread use of GPS time)
  - All satellite-based techniques (SLR, GPS, DORIS) use TT rather than TCG for coordinate time (introducing a scale of 0.7 ppb)
  - VLBI analysis conforms to this for consistency in determining the terrestrial reference frame (no effect on precision, only scale)

Light Time Computation

- Model for signal propagation differs in the two frames



Geocentric model needs only to account for Earth rotation during light time



Barycentric model must also account for orbital motion of Earth

"Relativistic Tides"

- Geocentric station coordinates must be transformed from the geocentric frame in which they are defined into the solar-system barycentric frame (contracted and scaled)

$$\mathbf{r}_B = \mathbf{r} - \left[\frac{\gamma\, U_E}{c^2} + L \right] \mathbf{r} - \frac{\mathbf{V}_E \cdot \mathbf{r}}{2c^2}\, \mathbf{V}_E$$

$\mathbf{r}$ = station position in the geocentric frame

$\mathbf{r}_B$ = station position in the barycentric frame

$\mathbf{V}_E$ = barycentric velocity of the Earth

U_E = Sun's gravitational potential at the Earth

L = scale difference between the frames

$(L \approx 1.4808 \times 10^{-8})$

Light-time Delay

- Space is curved in the vicinity of massive bodies, producing a delay in the measured light-time
- In the geocentric formulation, only the Earth's mass needs to be considered

Lageos laser ranging	-	Sun	18 cm
		Earth	1 cm
		Moon	ignored
Lunar laser ranging	-	Sun	7.6 m
		Earth	4 cm
		Moon	1 mm

Only need this for geocentric model

The form of the correction to be added to the computed (geometric) range ρ is given by:

$$\Delta\rho_{rel} = (1+\gamma)\frac{GM}{c^2}\ln\left[\frac{r_1 + r_2 + \rho}{r_1 + r_2 - \rho}\right]$$

The quantity r_1 is the distance between the station and the mass center, and r_2 is the distance between the satellite and the mass center.

Barycentric N-body Equations of Motion

$$
\begin{aligned}
\ddot{\mathbf{r}}_i = \sum_{j\neq i} \frac{\mu_j\left(\mathbf{r}_j-\mathbf{r}_i\right)}{{r_{ij}}^3}\Bigg\{ & 1-\frac{2(\beta+\gamma)}{c^2}\sum_{l\neq i}\frac{\mu_l}{r_{il}}-\frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} \\
& +\gamma\left(\frac{\dot{s}_i}{c}\right)^2+(1+\gamma)\left(\frac{\dot{s}_j}{c}\right)^2-\frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j \\
& -\frac{3}{2c^2}\left[\frac{\left(\mathbf{r}_i-\mathbf{r}_j\right)\cdot\dot{\mathbf{r}}_j}{r_{ij}}\right]^2+\frac{1}{2c^2}\left(\mathbf{r}_j-\mathbf{r}_i\right)\cdot\ddot{\mathbf{r}}_j\Bigg\} \\
& +\frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{{r_{ij}}^3}\left\{\left[\mathbf{r}_i-\mathbf{r}_j\right]\cdot\left[(2+2\gamma)\,\dot{\mathbf{r}}_i-(1+2\gamma)\,\dot{\mathbf{r}}_j\right]\right\}\left(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j\right) \\
& +\frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}}
\end{aligned}
$$

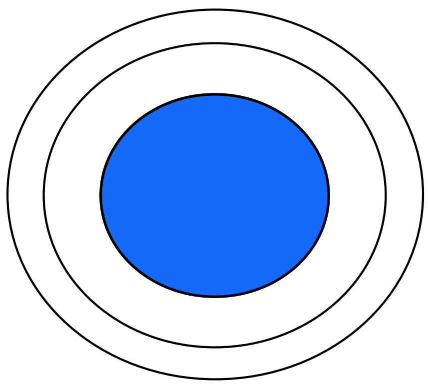
in General Relativity,
$\beta \equiv \gamma \equiv 1$

(non-spherical terms & Lense-Thirring precession would be included if higher accuracy is required)

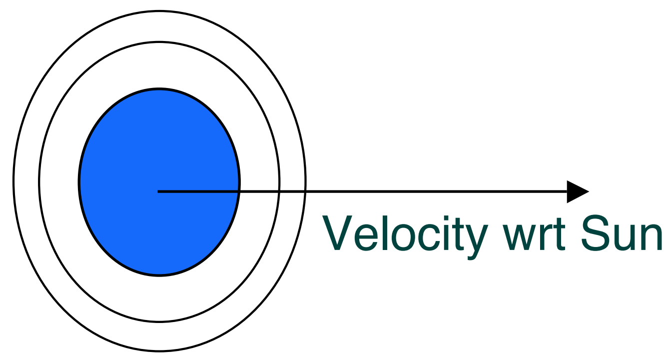
Relativistic Geopotential Correction for Barycentric Model

We found that the barycentric equations of motion were inadequate (large 280-day periodic errors for LAGEOS long-arcs)

The Earth's non-spherical gravity field (in the geocentric frame) must also be transformed into the barycentric frame in the same way as the geocentric station coordinates (*i.e.,* "relativistic tides")



Geopotential in geocentric frame



Geopotential in solar-system barycentric frame

Geocentric 1-body Relativistic Equations of Motion

Only the Earth needs to be considered in the geocentric formulation

$$\ddot{\mathbf{r}} = \frac{GM_E}{c^2 r^3}\Big[\, [\, (2\beta + 2\gamma)\,\frac{GM_E}{r} - \gamma(\dot{\mathbf{r}} \bullet \dot{\mathbf{r}})\,]\,\mathbf{r} + (2+2\gamma)\,(\mathbf{r} \bullet \dot{\mathbf{r}})\,\dot{\mathbf{r}}$$ Schwarzschild solution

$$+\,2\,(\boldsymbol{\Omega} \times \dot{\mathbf{r}})$$ Geodesic precession

$$+\;(1+\gamma)\frac{GM_E}{c^2 r^3}\left\{\frac{3}{r^2}\big[\,\mathbf{r} \times \dot{\mathbf{r}}\,\big]\big[\,\mathbf{r} \bullet \mathbf{J}\,\big] + \big[\,\dot{\mathbf{r}} \times \mathbf{J}\,\big]\right\}$$ Lense-Thirring precession

where

$$\boldsymbol{\Omega} \cong \left(\frac{1}{2} + \gamma\right)\left(\dot{\mathbf{R}} \times \left[\frac{-GM_S\;\mathbf{R}}{c^2 R^3}\right]\right)$$

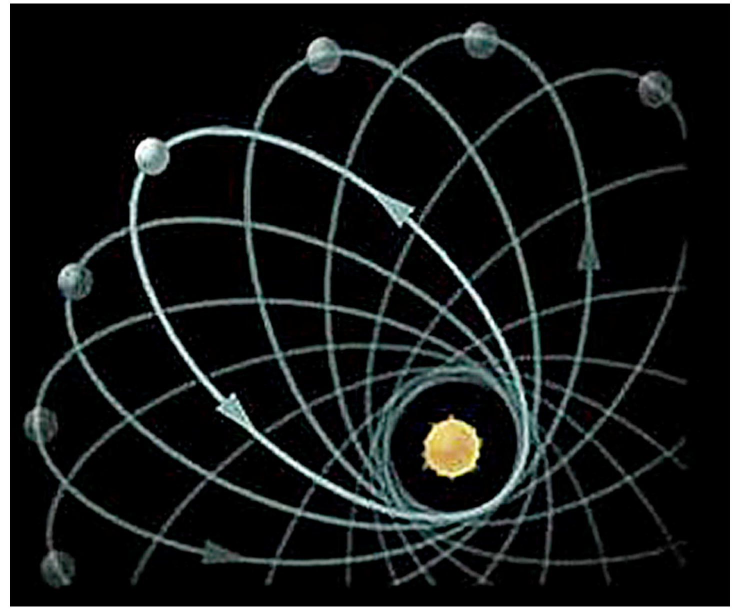
and $\mathbf{J}$ is the Earth's angular momentum per unit mass ($|\mathbf{J}| \cong 9.8 \times 10^8\ \mathrm{m^2/sec}$).

General Relativity and SLR Analysis

- Adopting geocentric reference frame for SLR analysis is adequate given current level of modeling and observation accuracy
  - 8-9 mm fits for 60-day arcs and 6-7 mm for 7-day arcs demonstrated for best stations (2-4 mm precision)
  - Should reassess this conclusion if mm-level performance is achieved (but likely to still be adequate for most satellites)
- GR impacts terrestrial reference frame (TRF) mainly through scale (estimation of GM_E and station heights)
- Precessions are interesting observable orbital effects

Perigee Precession

- “Non-linear” correction to two-body acceleration causes an advance of the periapse
- Explaining the excess perihelion advance of Mercury was an early confirmation of General Relativity

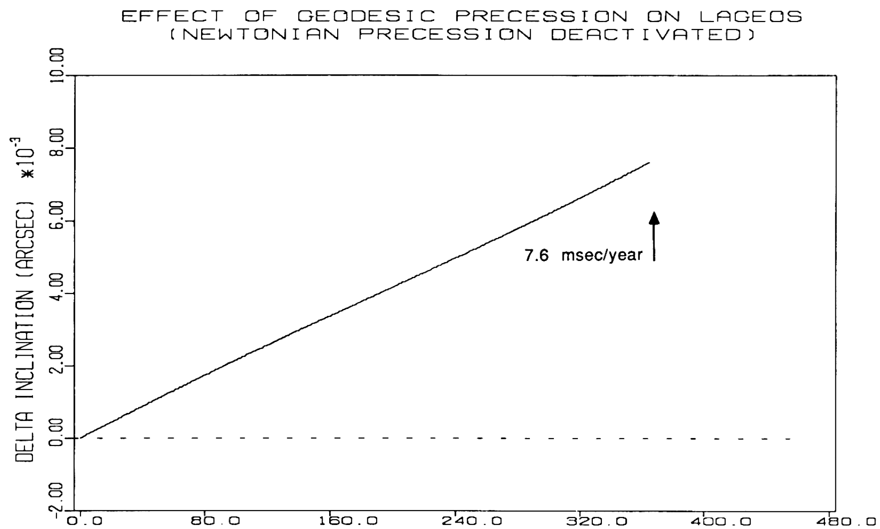


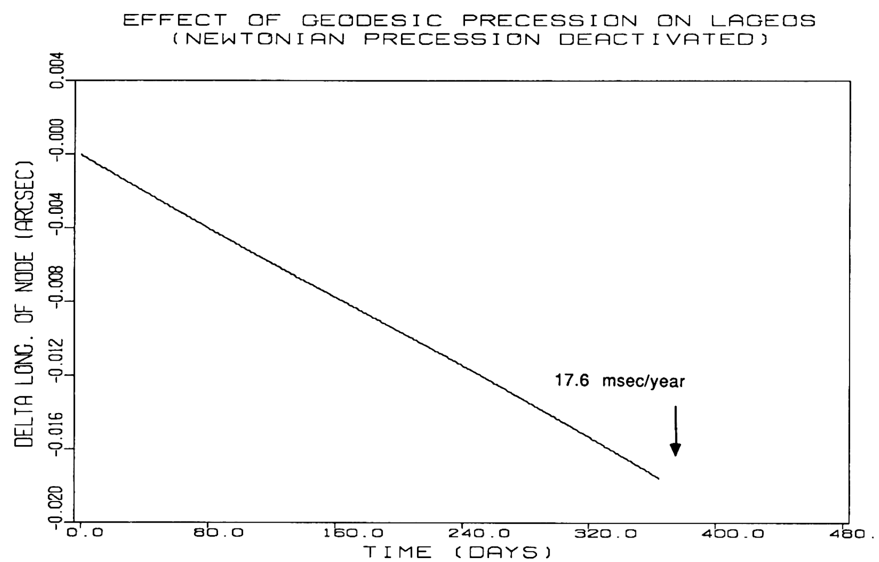
$$\Delta\ddot{\vec{r}} = \frac{GM_E}{c^2 r^3}\left\{\left[2(\beta+\gamma)\frac{GM_E}{r} - \gamma\dot{\vec{r}}\cdot\dot{\vec{r}}\right]\vec{r} + 2(1+\gamma)(\vec{r}\cdot\dot{\vec{r}})\dot{\vec{r}}\right\}$$

- For LAGEOS, the perigee precession from relativity of 9 mas/day is approximately 100 times larger than that due to the uncertainty in our knowledge of J2

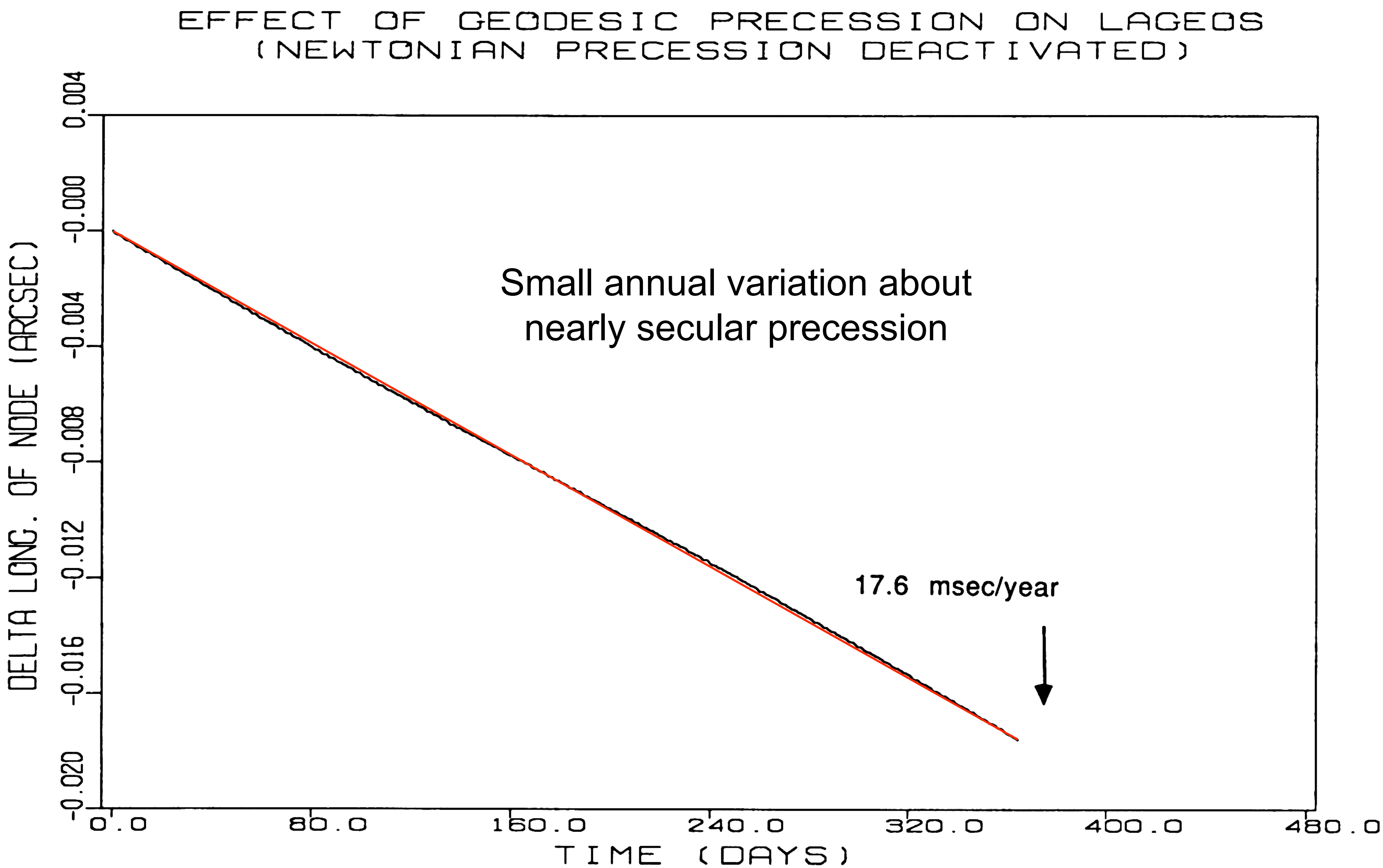
Geodesic Precession (1)

- The geocentric frame is defined as non-rotating relative to distant stars
- However, relativity predicts a precession of any inertial frame as it transverses the curved space-time around a massive body
- Since the frame itself is not allowed to precess, the satellite orbit appears to precess relative to the geocentric frame
- For the Earth’s orbit, this is 19.2 marcsec/yr about the ecliptic, which projects into inclination (7.6 mas/y) and node (17.6 mas/y) of the satellite orbit





Geodesic Precession (2)



'Frame Dragging'

- The effect of the rotating mass of the Earth on a satellite orbit is to 'drag' the satellite orbit plane in the direction of rotation (Lense-Thirring precession)
  - The most observable effect is on the node

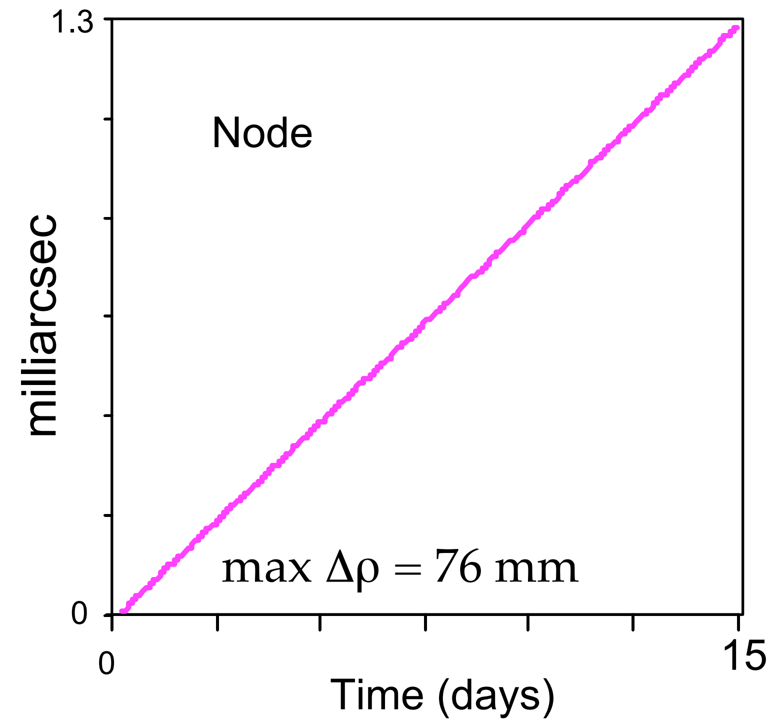
$$\dot{\Omega} = \frac{2G}{c^2 a^3} \frac{J}{(1-e^2)^{\frac{3}{2}}}$$ ≈ 31 marcsec/yr for LAGEOS

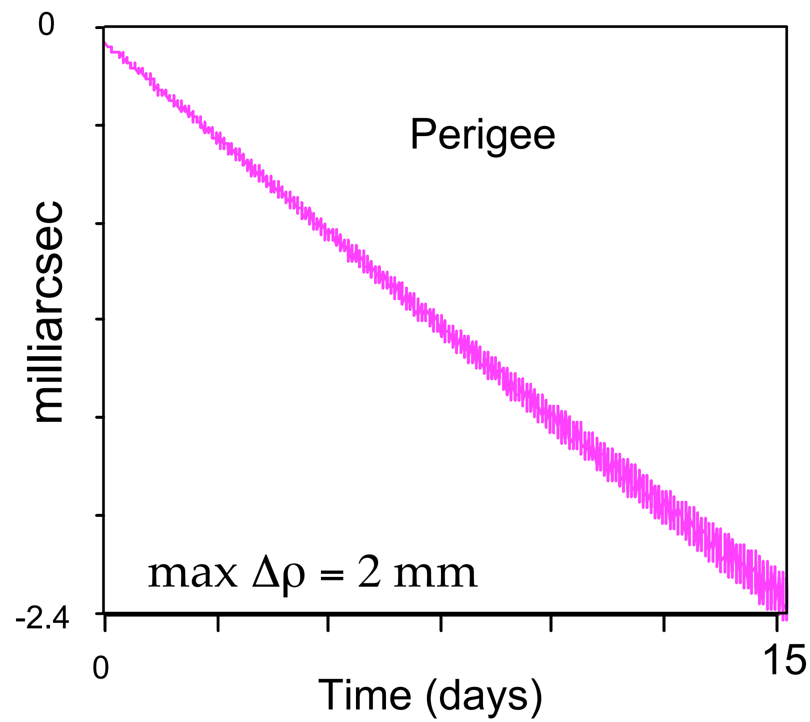
  - Can this 'pure' test of 'frame dragging' be observed in satellite orbits?

- A similar effect is predicted for an orbiting gyroscope (Schiff precession)
  - Gravity Probe-B mission was conceived and flown to test this prediction by General Relativity

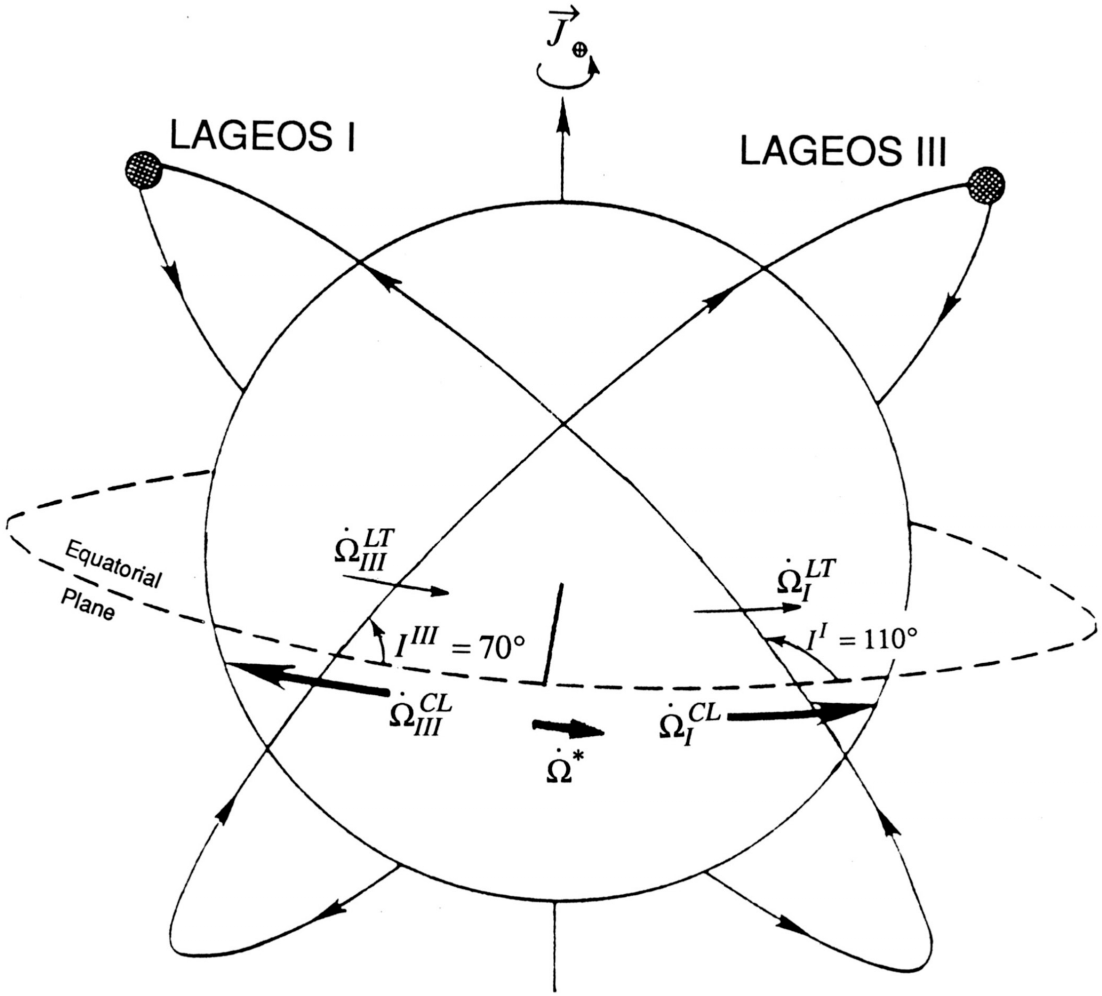
Lense-Thirring Effect

Effect of Lense-Thirring precession on Node and Perigee of LAGEOS-2 over 15 days





LAGEOS-3 Proposal



Object of measurement:

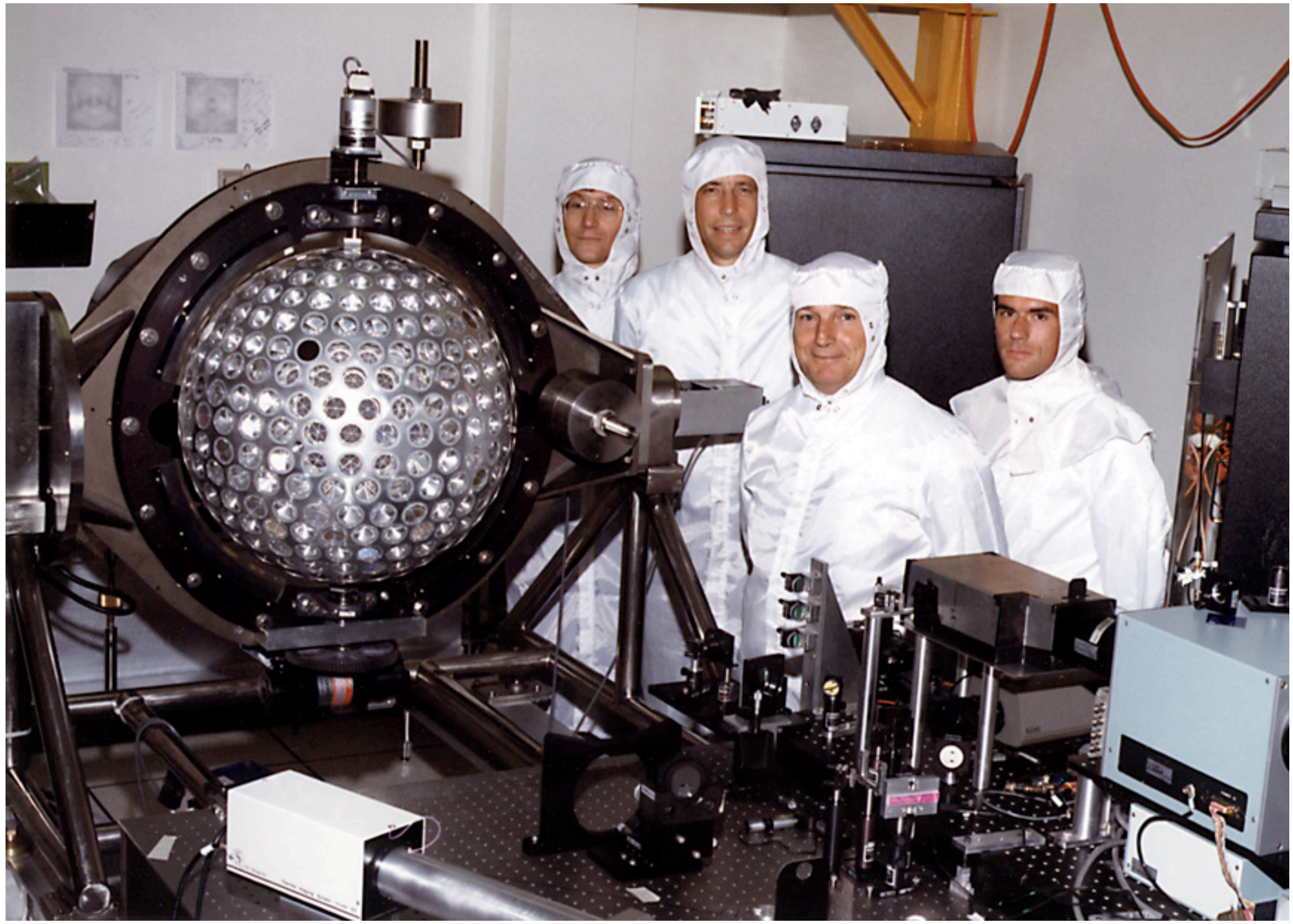
$$\dot{\Omega}^* = ½ (\dot{\Omega}^I + \dot{\Omega}^{III})$$

- LAGEOS-1 alone was insufficient because LT precession could not be separated from the much larger precession due to even zonal harmonics
- In 1989, it was proposed to launch a third satellite into an orbit with the same altitude as LAGEOS-1 but with opposite inclination
- This would cancel out effect of errors in all even zonal harmonics, and was estimated to provide a better than 10% test
- Unfortunately, not funded

Why Not Use LAGEOS-2?

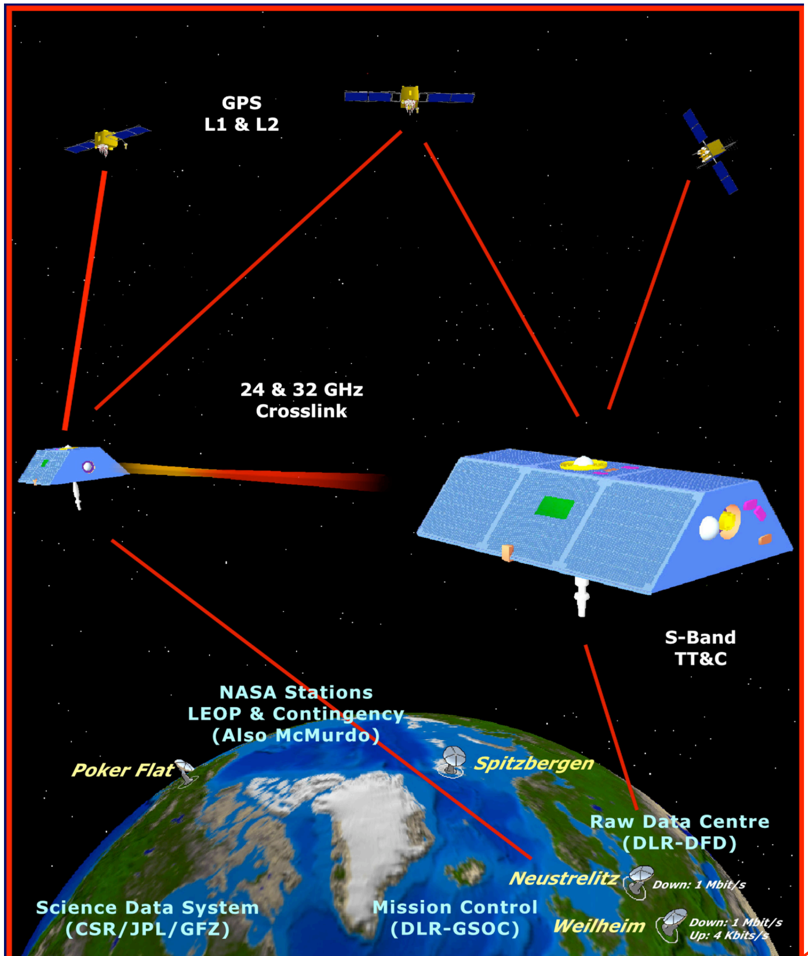
During this time, LAGEOS-2 was being prepared for launch

However, the orbit inclination chosen (52.6°) was not suitable (at the time) because the gravity model errors were too large



LAGEOS-2 at NASA/GSFC for optical testing
(left to right: J. Ries, R. Eanes, B. Tapley and M. Watkins)

Gravity Recovery and Climate Experiment (GRACE)



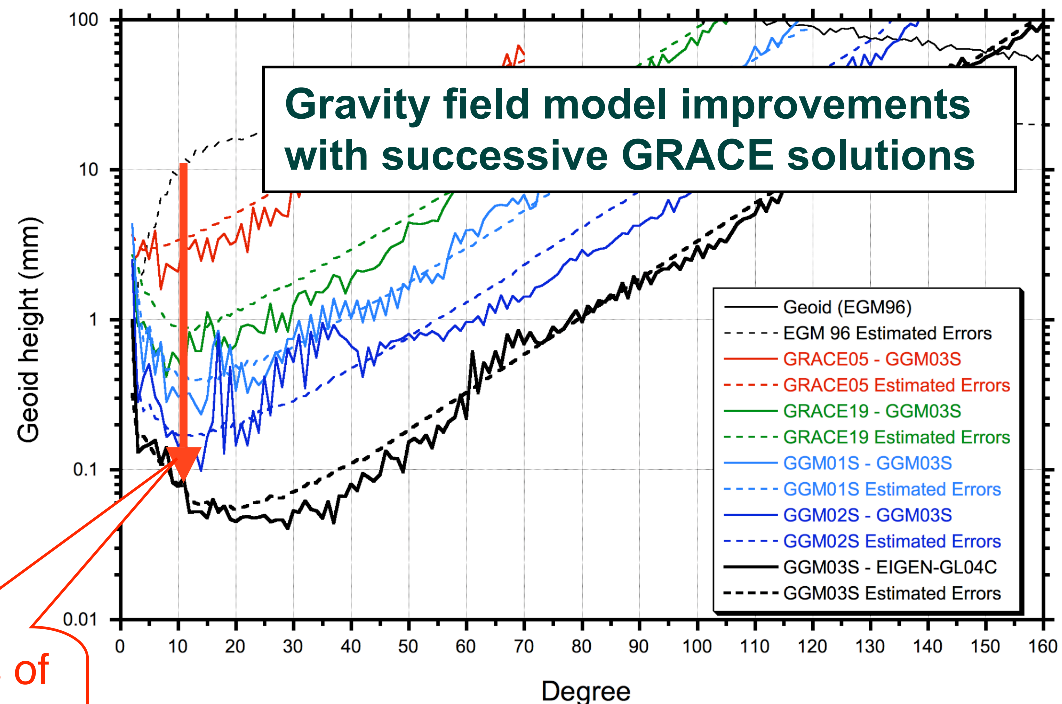
Joint NASA/DLR Mission

Launched: March 17, 2002

Altitude: ~460 km (-10 m/day)

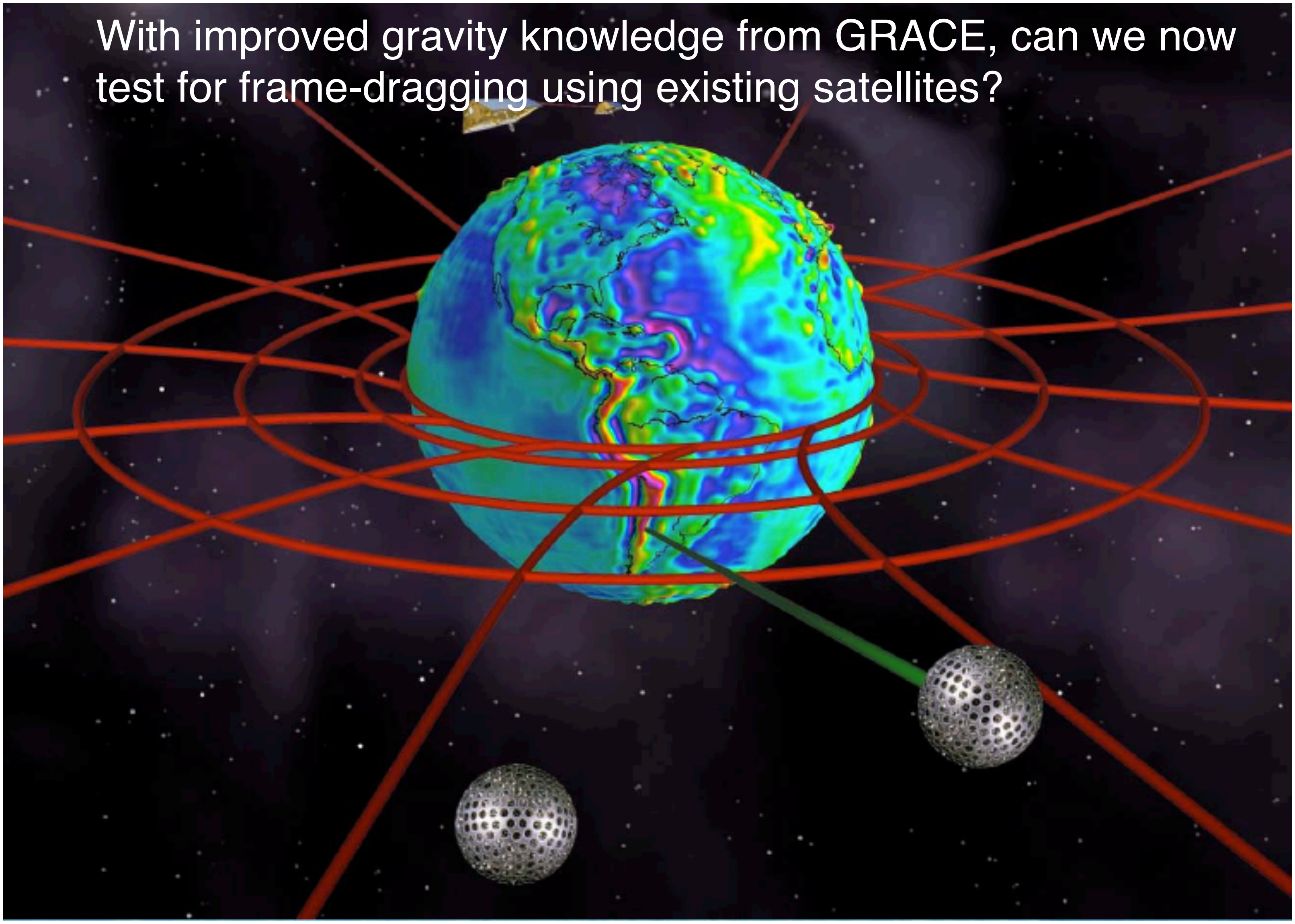
Inclination: 89 deg

Separation Distance: ~220 km



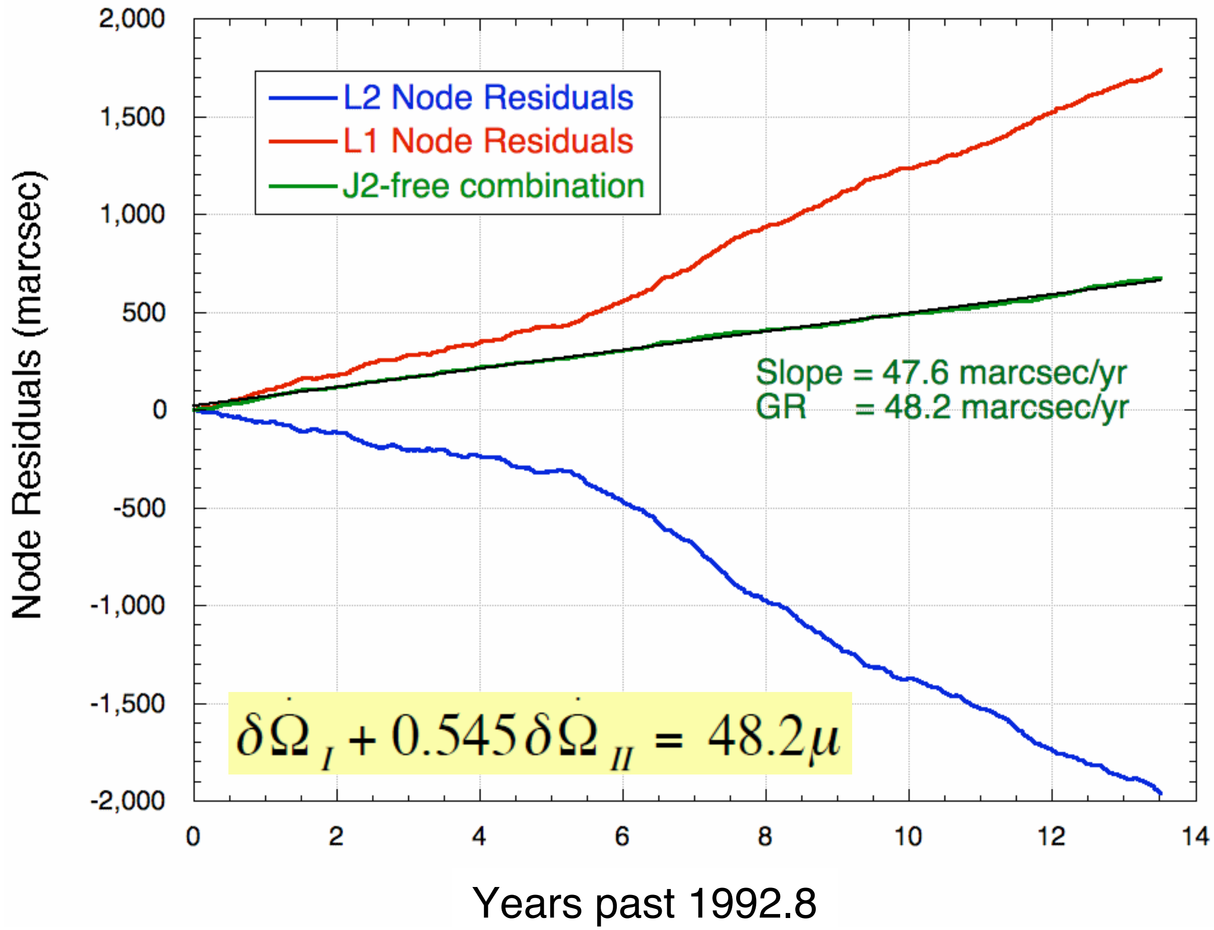
Two orders of magnitude improvement

LAGEOS-1/2 and GRACE



J2-Free Combination from LAGEOS-1 and LAGEOS-2

- Ciufolini and Pavlis (Nature, 2004) used EIGEN-GRACE02S to claim confirmation of GR prediction to ~10%.
- Using a more recent gravity solution from CSR and using 13.5 years of SLR data, we recovered GR value of LT precession to ~1%
- How reliable are these results?

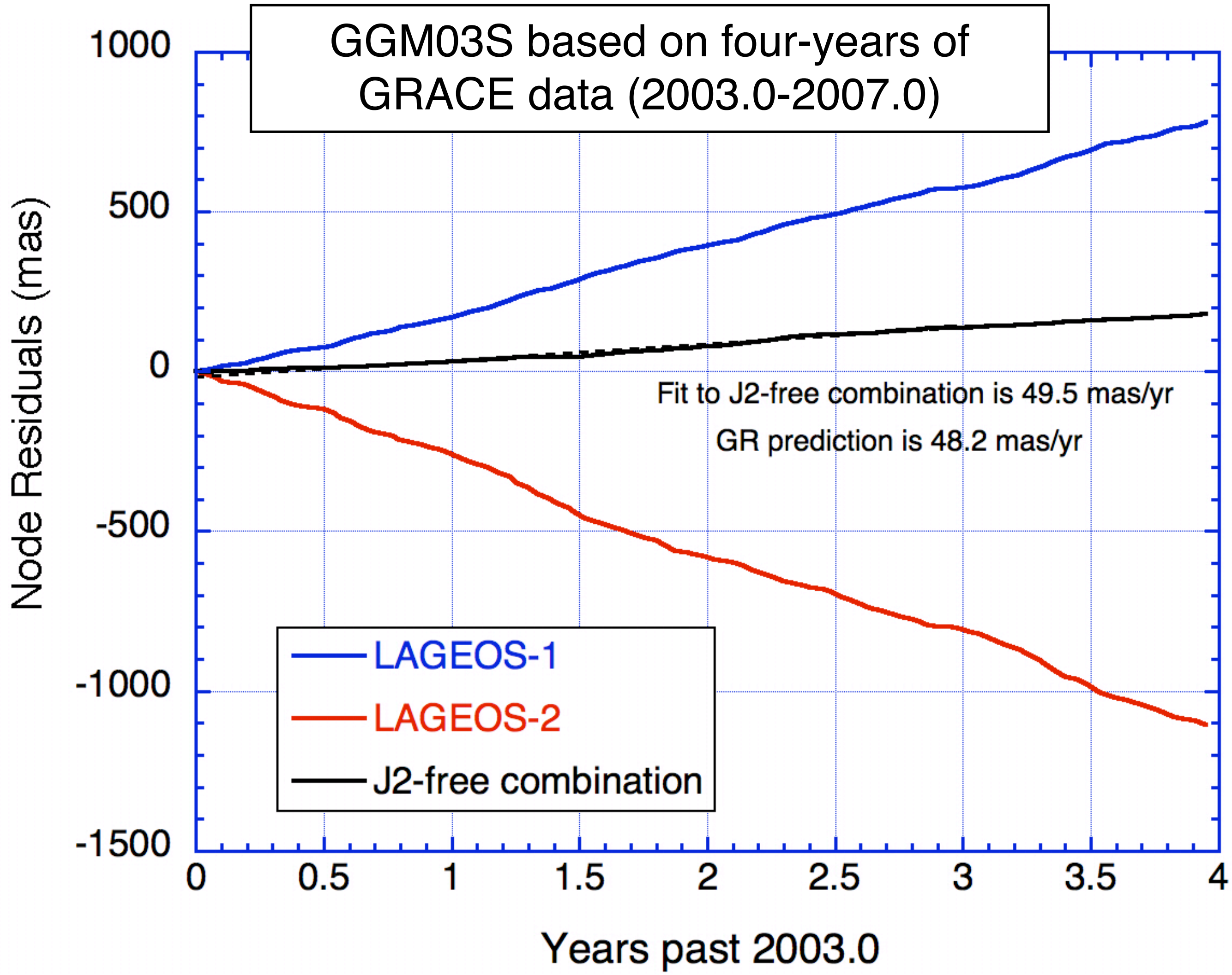


Note how large changes in the node series (due to significant changes in J2) cancel out in J2-free combination

LT Experiment over GRACE Mission only

An important concern in the error is the mapping of the even zonals from the <u>mean</u> epoch of the GRACE data to the <u>mean</u> epoch of the SLR data

Here we show an experiment just for the 4 years used for GGM03S



Gravity Model Uncertainty and LT Error

Recent GRACE gravity models

Gravity model	Year	LT signal / GR	C40	C40 Sigma	C60	C60 Sigma
EIGEN-GRACE02S	2004.1	1.25	5.40007101E-07	4.2E-13	-1.49930405E-07	9.6E-12
GGM02S	2004.6	1.01	5.39975648E-07	8.3E-12	-1.49939959E-07	4.5E-12
EIGEN-CG03C	2005.3	1.03	5.39987470E-07	3.8E-12	-1.49955461E-07	3.0E-13
GIF22a	2005.7	0.99	5.39989338E-07	1.5E-13	-1.49953540E-07	1.0E-13
JEM04G	2005.9	0.84	5.39970358E-07	1.2E-13	-1.49967559E-07	9.1E-14
EIGEN-GL04C	2006.3	0.93	[illegible]	[illegible]	-1.49953685E-07	1.8E-12
JEM01-RL03B	2006.9	1.05	[illegible]	[illegible]	-1.49956879E-07	6.2E-14
GGM03S	2007.5	0.85	[illegible]	[illegible]	-1.49959620E-07	1.6E-12
ITG-GRACE03S	2007.8	0.85	[illegible]	[illegible]	-1.49953913E-07	1.7E-13
GGM03S (2003-2007 only)	2007.5	1.03	5.39972911E-07	4.6E-12	-1.49959620E-07	1.6E-12
EIGEN-GRGS.RL02 (2003-2007 only)	2009.3	0.98	5.39990383E-07	3.8E-13	-1.49953448E-07	1.7E-13
Mean		0.99	5.39981642E-07		-1.49953099E-07	
StDev		0.12	1.3E-11		1.0E-11	

Four-year tests within 2-3% of GR

Mean within 1% of GR, scatter of 12%

Other 'sanity' tests

GGM02S (model LT)	0.01	(differs by exactly 1.0 as expected)
GGM02S (no GP)	1.58	(Geodesic precession ~57% of LT)
GGM02S (no rates for J3,J4,J6)	1.02	(quadratic from rates is negligible)

Assigned errors (from covariance) for J4 and J6 are probably optimistic;
We can use scatter of estimates to arrive at a more realistic error estimate

Estimated Error Budget for LT Test

Error Source	% of LT
Scatter due to method (linear fit w/wo tidal lines)	1
Solar radiation pressure, Earth albedo, thermal reradiation effects	3
Zonal rates (quadratic effect; after mapping to mean epoch) *	1
J4 (estimated from scatter of GRACE gravity models) **	10
J6 (estimated from scatter of GRACE gravity models) **	5
J4-dot (20% uncertainty in mapping to mean epoch) ***	2
J6-dot (50% uncertainty in mapping to mean epoch) ***	2
RSS (% of LT)	12

* Epoch of GRACE gravity models typically ~2004.0; mean epoch of SLR data ~2000.0

** Assigned sigmas typically too small; used J4 scatter 1.3e-11, J6 scatter 1.2e-11

*** J4-dot uncertainty is estimated to be 20% of -1.4e-11/yr; 50% of 0.6e-11/yr for J6-dot

Tidal and seasonal variations cancel for J2 and have little effect on 13.5 year trend for other harmonics

Error estimate of ~12% consistent with scatter of LT estimates

Including a third satellite (LARES) can eliminate error from J4

Summary

- General Relativity is an important consideration in light of current SLR observation and modeling accuracy
  - Geocentric formulation is adequate for near-Earth satellites, resulting in simpler force and measurement modeling
- SLR to the two LAGEOS satellites appears to confirm GR's prediction of the Lense-Thirring precession to better than 15% (conservatively)
  - This is made possible with improved geopotential models from the GRACE mission
  - Uncertainties in J4 and J6 still dominate current error budget
- LARES will allow a J2-J4-free combination
  - Uncertainty in J6 will be reduced further with future GRACE models