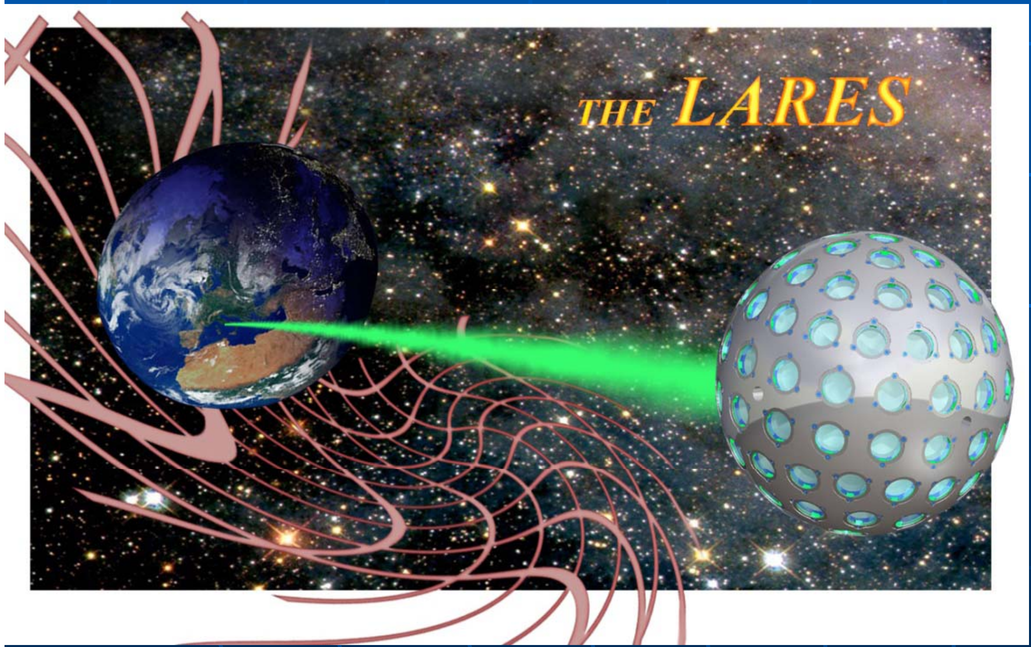
Testing General Relativity with LARES, LAGEOS and LAGEOS 2 (a heritage of John Archibald Wheeler)

ONE YEAR AGO PASSED AWAY JOHN ARCHIBALD WHEELER ONE OF THE MASTERS OF PHYSICS OF XX CENTURY AND FATHER OF THE REINASSANCE OF GENERAL RELATIVITY



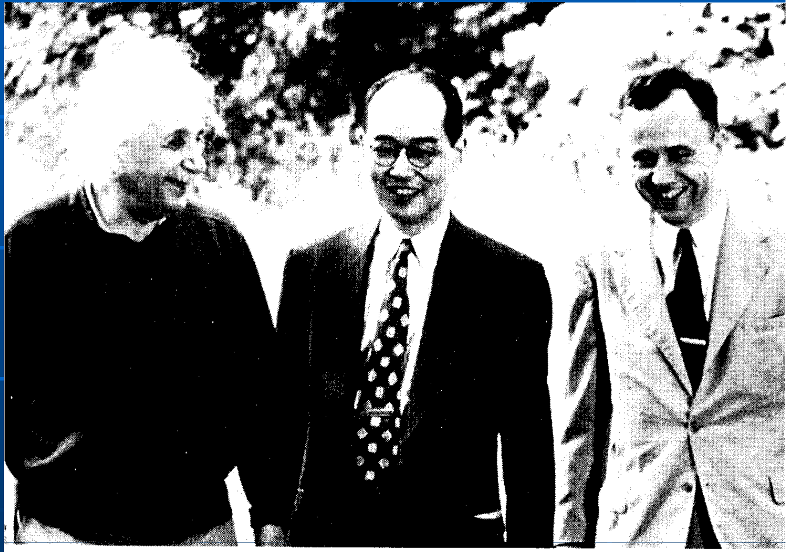
Ignazio Ciufolini
(Univ. Salento)
Rome 3-July-2009

- International School on Astrophysical Relativity

«John Archibald Wheeler»

- at EMFCSC (*Erice, Italy: 2006, 2008*)

John's office,
Univ. Texas at Austin,
nearly 20 years ago



Walking in Princeton with Albert Einstein and Hideki Yukawa, 1953.
(Photograph by Howard Schrader, courtesy of Princeton University.)

ACCURATE MEASUREMENT OF GRAVITOMAGNETISM WITH THE LARES AND LAGEOS SATELLITES

BRIEF INTRODUCTION

EXPERIMENTS

- Main experimental efforts to measure frame-dragging
- The 1995-2003 observations using the LAGEOS satellites
- The 2004-2007 accurate measurements using the GRACE Earth's gravity models and the LAGEOS satellites
- LARES: 2010

Ignazio Ciufolini (Univ. Salento): Rome-3-7-2009

DRAGGING OF INERTIAL FRAMES

(*FRAME-DRAGGING* as Einstein named it in 1913)

- **The local inertial frames are dragged by mass-energy currents: $\varepsilon\, u^{\alpha}$**

  $$G^{\alpha\beta} = \chi\, T^{\alpha\beta} =$$

  $$= \chi\, [(\varepsilon + p)\, u^{\alpha} u^{\beta} + p\, g^{\alpha\beta}]$$

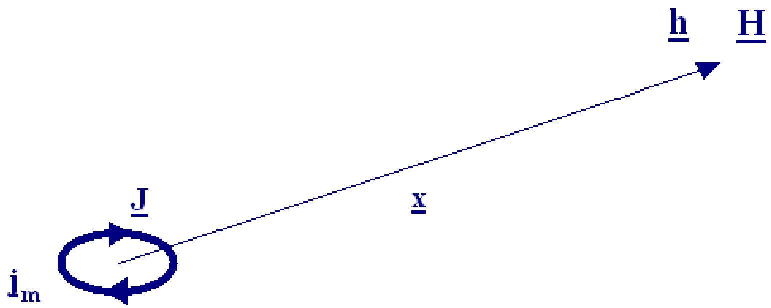
- It plays a key role in high energy astrophysics

  (Kerr metric)

Thirring 1918
Braginsky, Caves and Thorne 1977
Thorne 1986
Jantsen et al. 1992-97, 2001
I.C. 1994-2001

THE WEAK-FIELD AND SLOW MOTION ANALOGY WITH ELECTRODYNAMICS

Gravitomagnetic Field in General Relativity



From weak field and slow motion limit of G= χ T:

$$\Delta\ h_{0i}\ \cong\ 16\ \pi\ \rho\ v^i$$ Lorentz gauge

Electromagnetism

$$\Delta\, \mathbf{A}\ = -\frac{4\pi}{c}\ \mathbf{j}$$

where $\mathbf{h} \equiv (h_{01},\ h_{02}\ , h_{03})$ is the gravitomagnetic potential

$$h_{0i}\ (\mathbf{x})\ \cong\ -\ 4\int \frac{\rho(\mathbf{x}')v^i(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\ d^3x'$$

$$\mathbf{A}\,(\mathbf{x})\ =\ \frac{1}{c}\int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\ d^3x'$$

$$\mathbf{h}\ (\mathbf{x})\ \cong\ -2\ \frac{\mathbf{J}\ \times\ \mathbf{x}}{|\mathbf{x}|^3}$$

$$\mathbf{A}\,(\mathbf{x})\ \cong\ \frac{\mathbf{m}\ \times \mathbf{x}}{|\mathbf{x}|^3}$$

The gravitomagnetic field is:

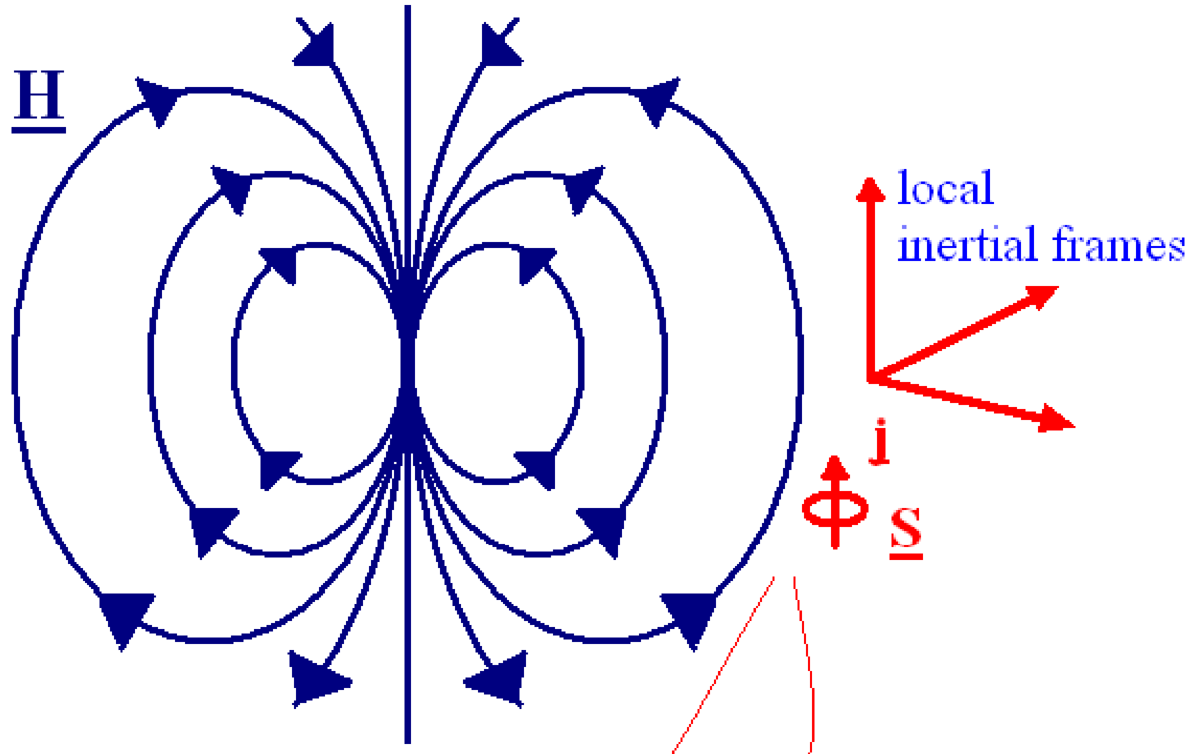
$$\mathbf{H}\ =\ \nabla\ \times\ \mathbf{h}\ \cong\ 2\left[\frac{\mathbf{J}-3(\mathbf{J}\cdot\hat{\mathbf{x}})\ \hat{\mathbf{x}}}{|\mathbf{x}|^3}\right]$$

$$\mathbf{B}\ =\ \nabla\ \times\ \mathbf{A}\ \cong\ \cong\ \frac{3\ \hat{\mathbf{x}}\ (\hat{\mathbf{x}}\cdot\mathbf{m})\ -\ \mathbf{m}}{|\mathbf{x}|^3}$$

From weak field and slow motion limit of D u=0:

$$m\ \frac{d^2\mathbf{x}}{dt^2}\ \cong\ m\ (\mathbf{G}\ +\ \frac{d\mathbf{x}}{dt}\ \times\ \mathbf{H})$$

$$m\ \frac{d^2\,\mathbf{x}}{dt^2}\ =\ q\,(\mathbf{E}\ +\ \frac{d\mathbf{x}}{dt}\ \times\mathbf{B})$$



$$\mathbf{F} = \left(\frac{1}{2} \mathbf{S} \cdot \nabla \right) \mathbf{H} \qquad \tau \cong \tfrac{1}{2} \mathbf{S} \times \mathbf{H} = \frac{d\mathbf{S}}{dt} \equiv \dot{\mathbf{\Omega}} \times \mathbf{S}$$

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2} \mathbf{H} = \frac{-\mathbf{J} + 3\,(\mathbf{J} \cdot \hat{\mathbf{x}})\,\hat{\mathbf{x}}}{|\mathbf{x}|^3}$$

Dragging of inertial frames:

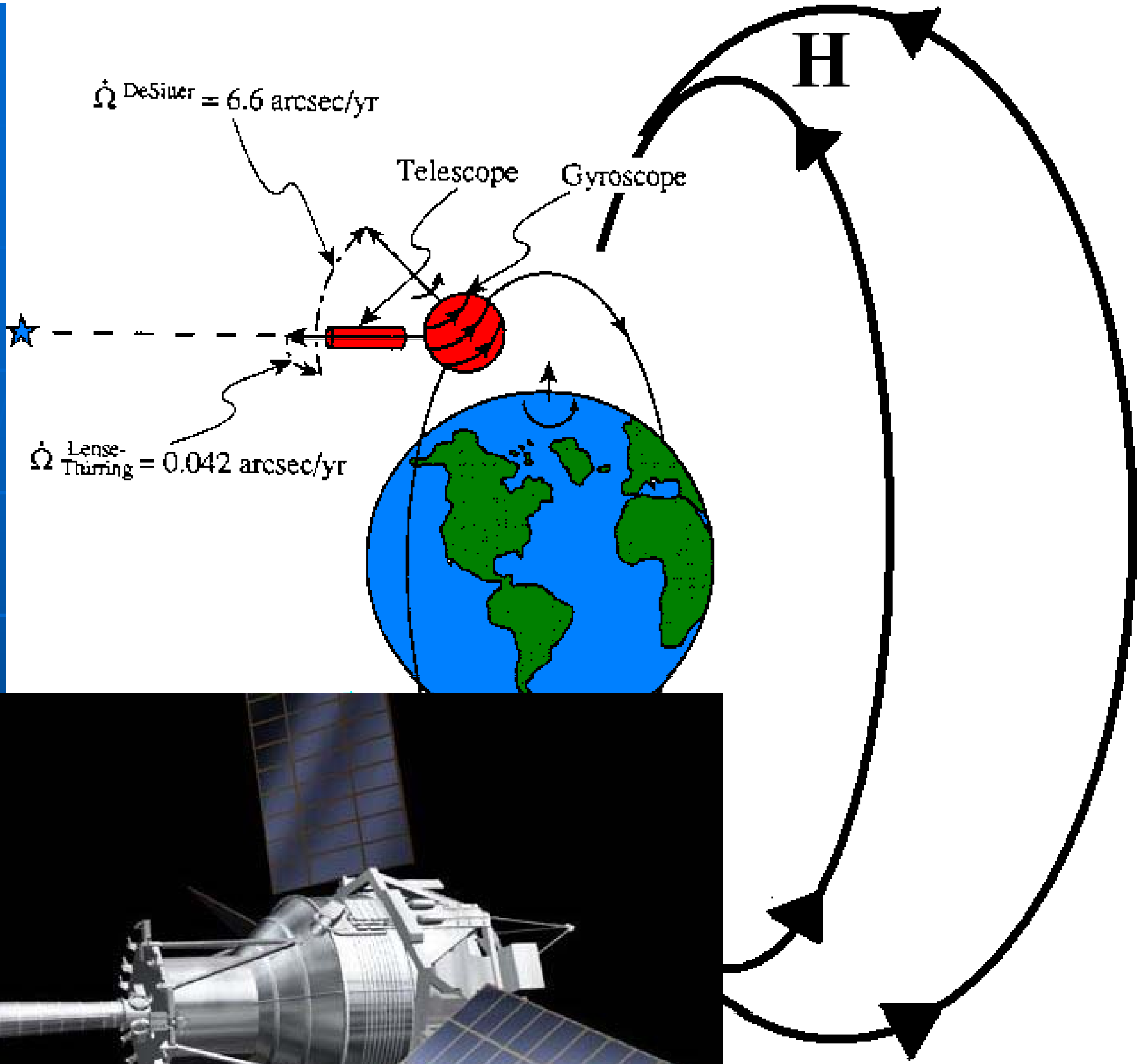
Mach principle in general relativity

GRAVITATION AND INERTIA

I.C. and J.A. Wheleer -1995

SOME EXPERIMENTAL ATTEMPTS TO MEASURE FRAME-DRAGGING AND GRAVITOMAGNETISM

- **1896: Benedict and Immanuel FRIEDLANDER**
  (torsion balance near a heavy flying-wheel)
- **1904: August FOPPL (Earth-rotation effect on a gyroscope)**
- **1916: DE SITTER (shift of perihelion of Mercury due to Sun rotation)**
- **1918: LENSE AND THIRRING (perturbations of the Moons of solar system planets by the planet angular momentum)**
- **1959: Yilmaz (satellites in polar orbit)**
- **1976: Graziani-Breakwell-Van Patten-Everitt**
  (two non-passive counter-rotating satellites in polar orbit: a very expensive experiment)
- **1960: Schiff-Fairbank-Everitt (Earth orbiting gyroscopes)**
- **1977-78: Cugusi and Proverbio, on LAGEOS only (however, wrong rate for frame-dragging)**
- **1986: I.C.: USE THE NODES OF TWO LAGEOS SATELLITES**
  (two supplementary inclination, passive, laser ranged satellites)
- **1988 : Nordtvedt (Astrophysical evidence from periastron rate of binary pulsar)**
- **1995-2007: I.C. et al. (obs. & measurements using LAGEOS and LAGEOS-II)**
- **1998: Some astrophysical evidence from accretion disks of black holes and neutron stars**
- **2004 launch of Gravity Probe B**
- **2010 LARES**



GRAVITY PROBE B

Problems with the GP-B data analysis have been outlined, see, for example: **R. F. O'Connell "Gravito-Magnetism in one-body and two-body systems: Theory and Experiment", in, "Atom Optics and Space Physics", Proc. of Course CLXVIII of the International School of Physics "Enrico Fermi", Varenna, Italy, 2007, ed. E. Arimondo, W. Ertmer and W. Schleich, (2009).**

ERROR: A WORK IN PROGRESS

G.M. Keiser, and J. Turneaure

Quantifying the Error

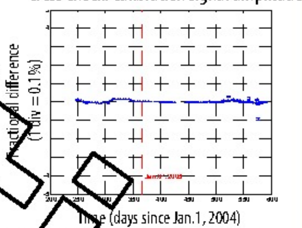
nsitivity Analysis
- ntentionally degrade data for one parameter
- erform full mission analysis with degraded data
- ompare degraded relativity result to original
- ifference provides experiment error for parameter

ro to gyro consistency
- onsistency increases confidence in result
- ombining consistent results decreases error

ernal cross checks build confidence
- tability of measured reference parameters

or tree to track/organize effort

Cross check: Calibration signal amplitude



finition: The residual relativity uncertainty associated with the modeling of ystematic effect is the systematic error of that effect.

Gyro Torques (I)

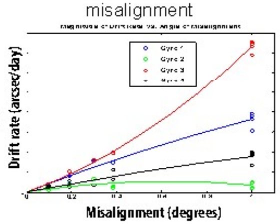
Misalignment torque: Torque proportional to angle subtended by SV roll axis and gyro spin axis

Cause: Interaction of patch fields on rotor and casing

Induced drift rate ~0.1 to 1.0 arcsec/yr

Require > 99 % removal for 10^{-3} arcsec/yr

Measured drift rate vs. misalignment



Misalignment torque: direction is know

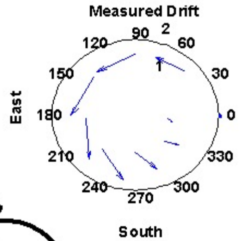
Mitigation: phase sensitive detection

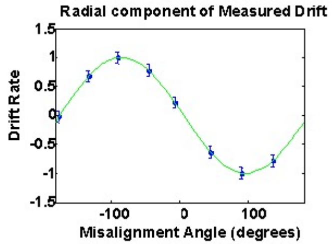
measure relativity in orthogonal direction

Error in resulting mitigated drift:

Uncertainty in torque direction: gives 4 marcsec/yr

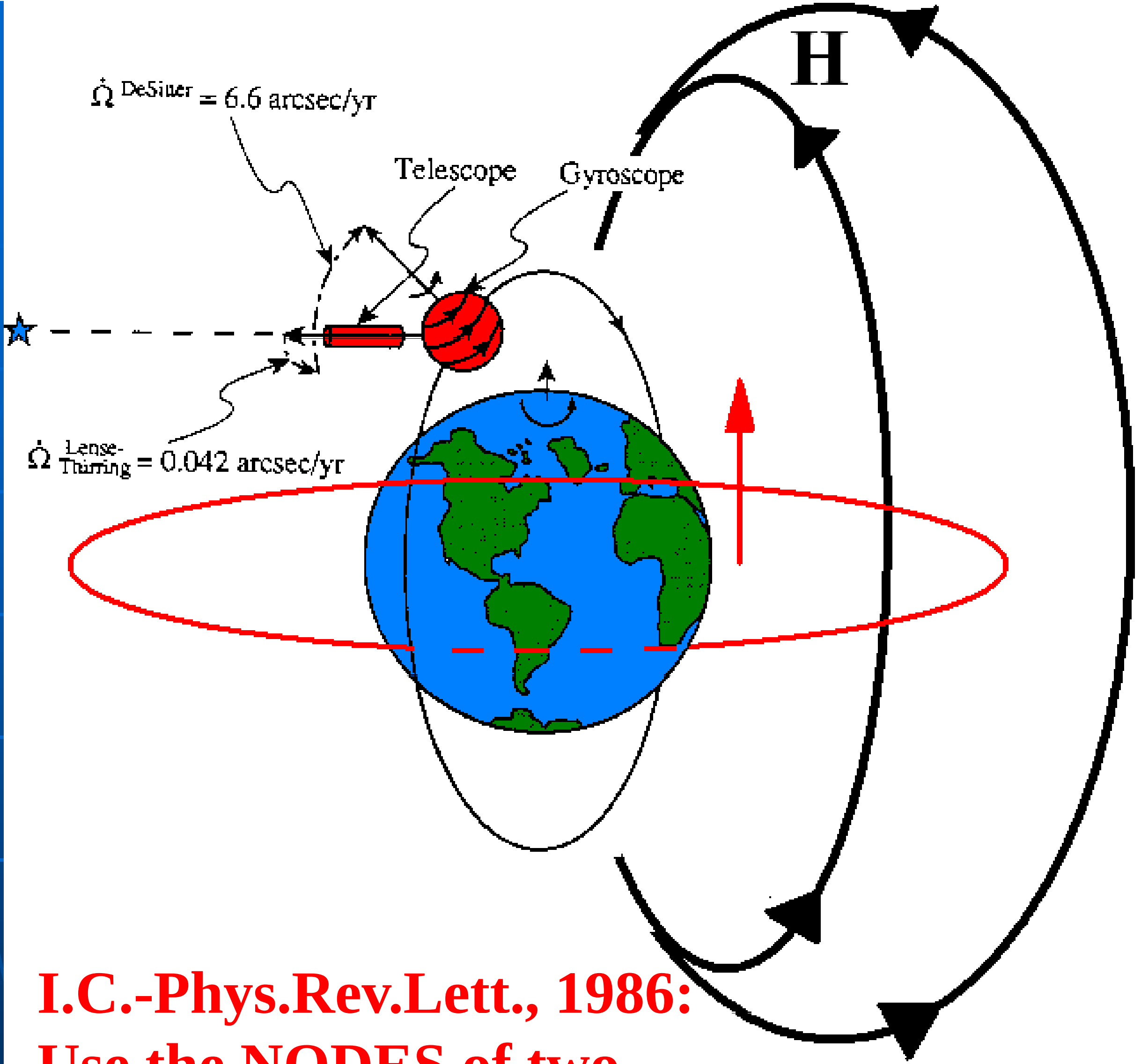
Changing amplitudes of misalignment, torque coefficient





Current Limits on Experiment Error

Error Source	Current Error (marcsec/yr)	Mitiga
Current error including resonance observation	100	Impro mode
EMI effects	5	Dat gradin nois remo
Misalignment patch-effect torque	4 *	Impro misali mer mod
Polhode frequency error	2.5	Trapp flux



I.C.-Phys.Rev.Lett., 1986:
Use the NODES of two LAGEOS satellites.

The proposal to use of the nodes of two laser-ranged satellites of LAGEOS type to measure the Lense-Thirring effect by eliminating in this way the Earth spherical harmonics uncertainties:

I.C. PRL 1986, I.C. I.J.M.P.D 1989, Tapley, Ries, Eanes, I.C., et al. NASA-ASI study 1989, J. Ries 1989, I.C. NCA 1996.

Calculation of the standard relativistic perigee precession of LAGEOS: Rubincam 1977

Proposal to use laser ranging to artificial satellites to observe relativistic effects,

among which the Lense-Thirring effect: Cugusi, Proverbio 1978 (the LAGEOS Lense-Thirring precession was calculated to be 4 arcsec/century, i.e. 40 milliarcsec/yr, instead of the correct 31 milliarcsec/yr figure calculated in I.C. and the problem of the Earth's even zonal harmonics errors was not treated)

Proposal to use polar satellites to solve the problem of the Earth's spherical harmonics:

Yilmaz 1959, Graziani, Breakwell, Van Patten, Everitt 1976

Proposal to use of the nodes of a number of laser ranged satellites ''... A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J2, J4, J6, etc, and one satellite to measure the Lense-Thirring effect: I.C 1989, I.C., J.A. Wheeler 1995

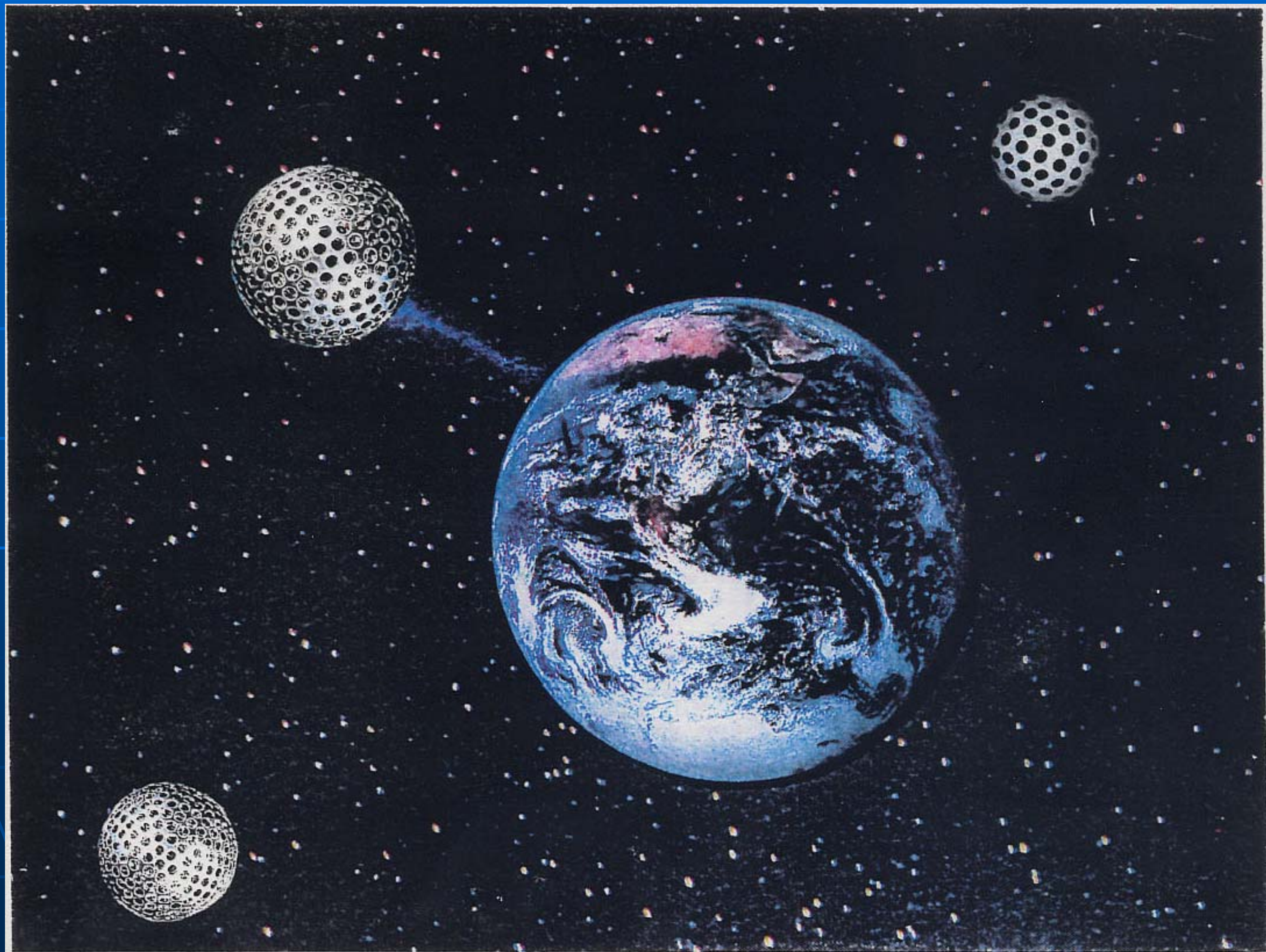
Proposal to use of the nodes of LAGEOS and LAGEOS 2, together with the explicit expression of

the LAGEOS satellites nodal equations: I.C. 1986

Detailed study of the various possibilities to measure the Lense-Thirring effect using LAGEOS and other laser-ranged satellites: Peterson 1997

Proposal to use of GRACE-derived gravitational models, when available, to measure the Lense-Thirring effect: Ries et al 1998 and E. Pavlis 2000

Satellite Laser Ranging



Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

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(Received 16 October 1984; revised manuscript received 19 April 1985)

We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

PACS numbers: 04.80.+z

In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ processes at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\mathbf{\Omega}} \times \mathbf{S}$ where

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \tag{2}$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} = d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static $-g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

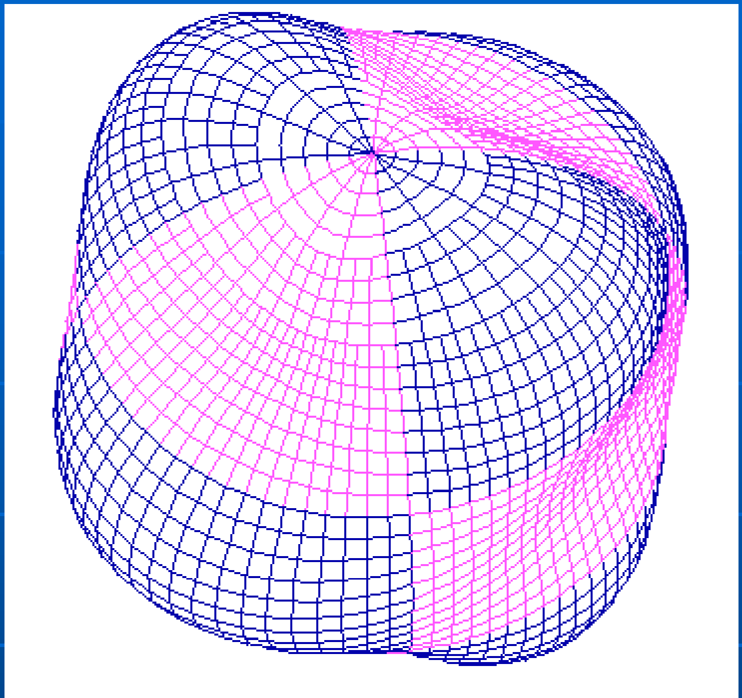
All these effects are quite small for an artificial satellite orbiting the Earth.

We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8-10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field—quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

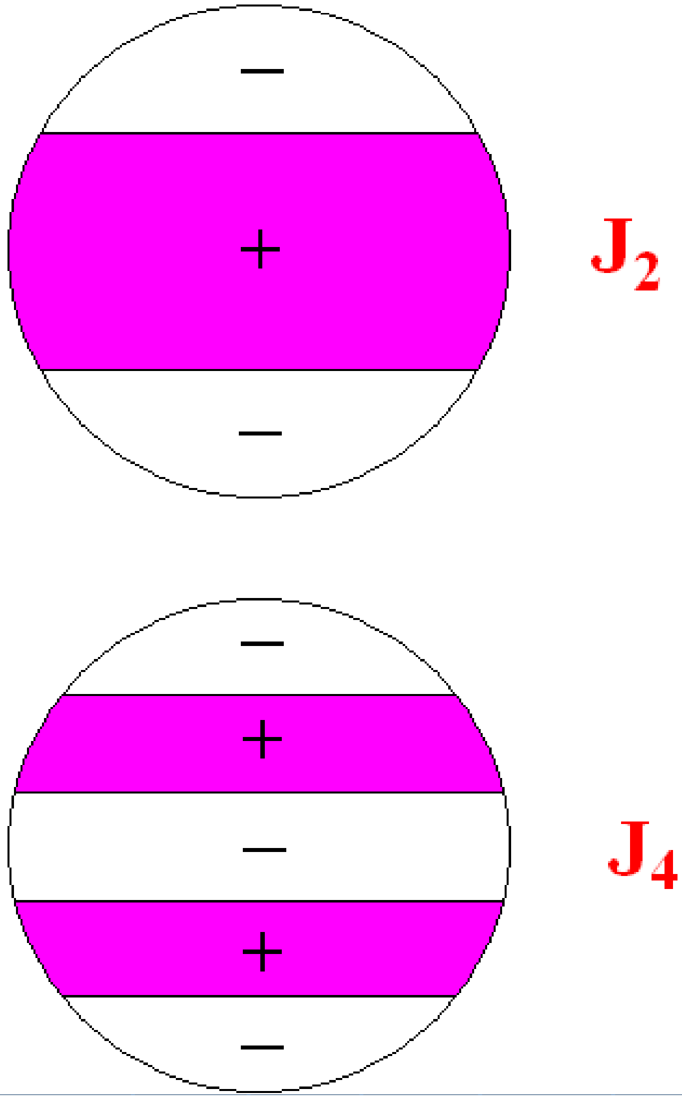
$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2\frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2(7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \dots\right\}, \tag{3}$$

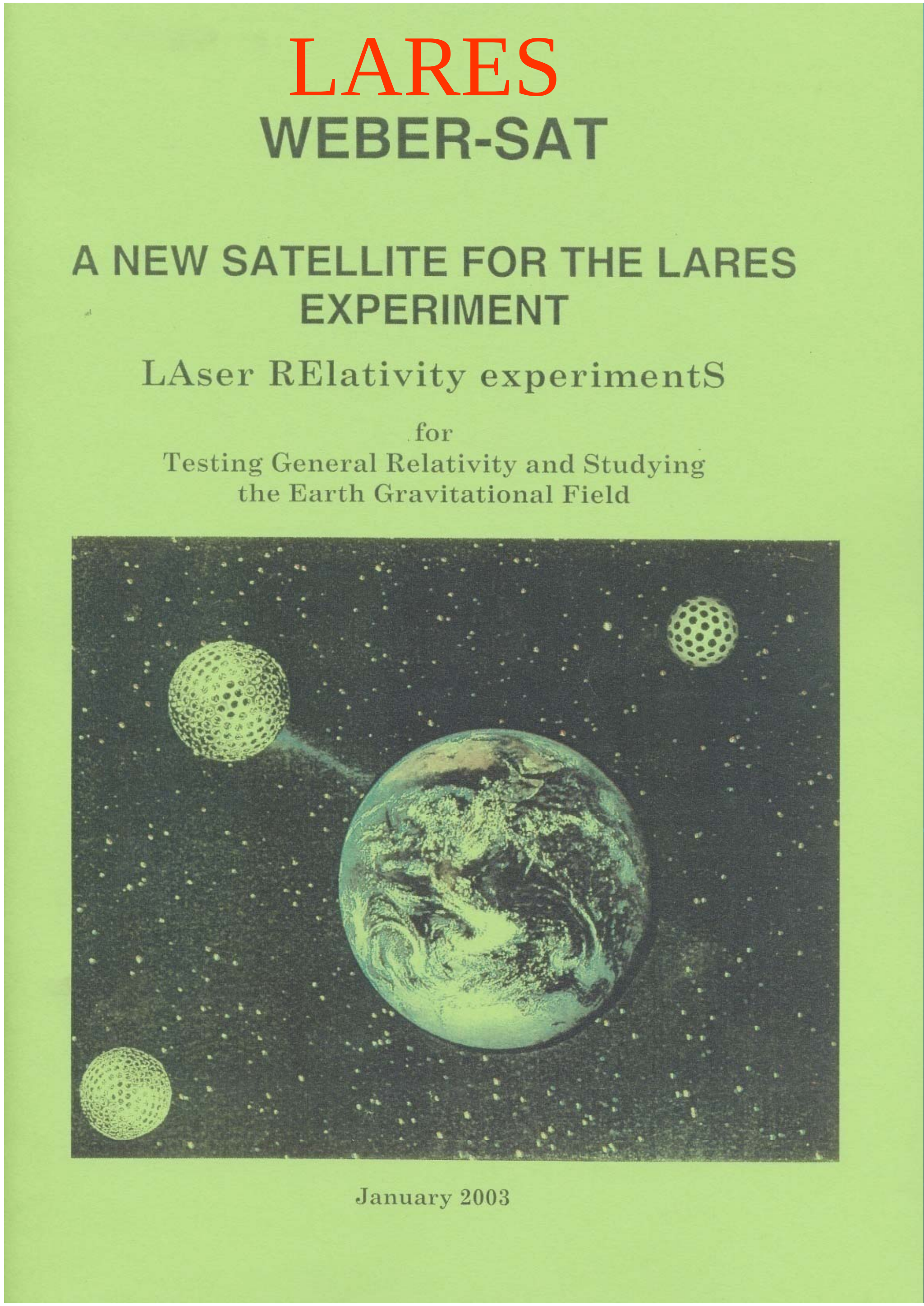
IC, PRL 1986:
Use of the nodes of two laser-ranged satellites to measure the Lense-Thirring effect



l=3, m=1

EVEN ZONAL HARMONICS




LARES

WEBER-SAT

A NEW SATELLITE FOR THE LARES EXPERIMENT

LAser RElativity experimentS

for

Testing General Relativity and Studying the Earth Gravitational Field

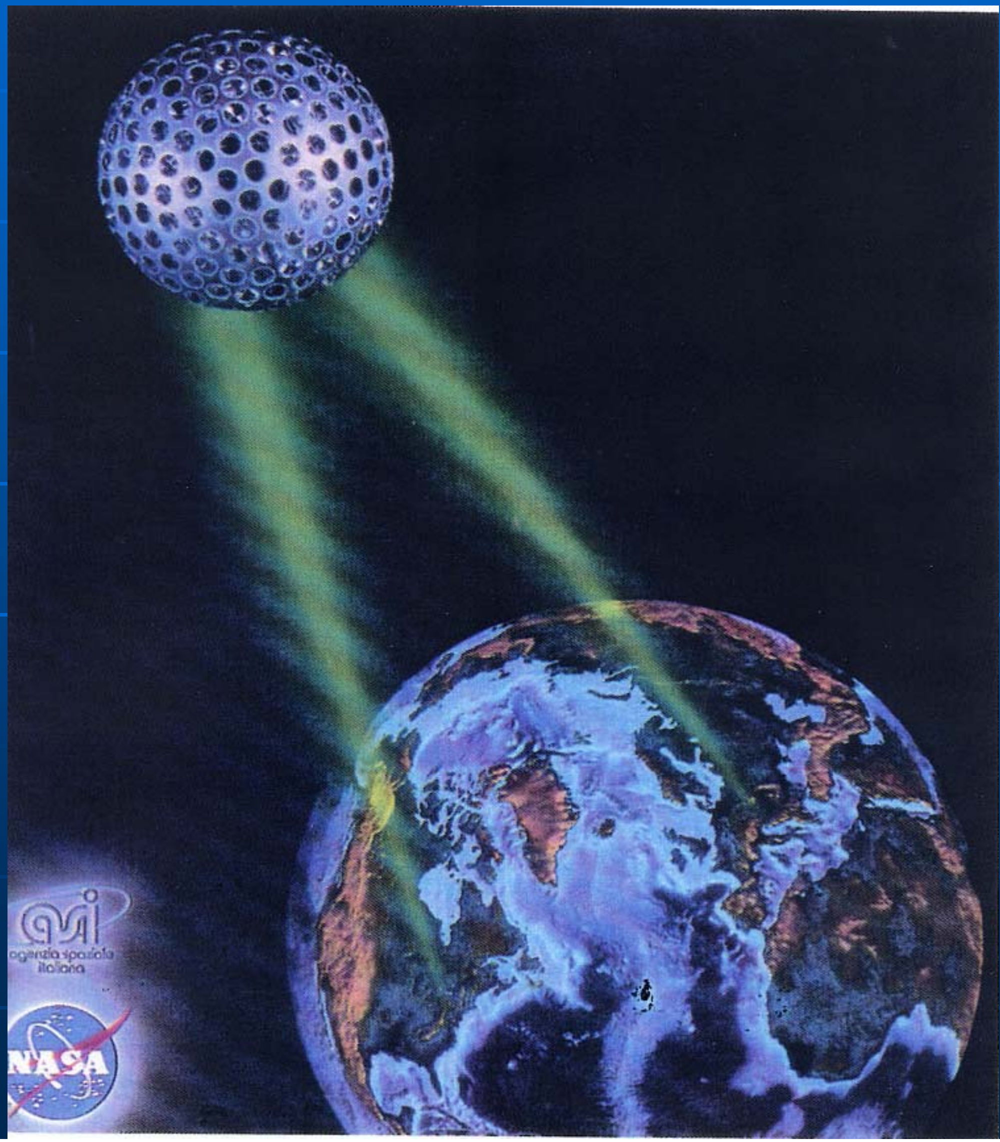
January 2003

MAIN COLLABORATION

- University of Lecce and INFN
- "Sapienza" University of Roma
- University of Maryland
- NASA-Goddard
- University of Texas at Austin
- GFZ-Potsdam/Munich (GRACE team)
- AstroSpace Center of
- Lebedev Phys. Inst.-Moscow

Lageos II: 1992

However, NO LAGEOS satellite with supplementary inclination to LAGEOS has ever been launched. Nevertheless, LAGEOS II was launched in 1992.



International Journal of Modern Physics A, Vol. 4, No. 13 (1989) 3083–3145

A COMPREHENSIVE INTRODUCTION TO THE LAGEOS GRAVITOMAGNETIC EXPERIMENT: FROM THE IMPORTANCE OF THE GRAVITOMAGNETIC FIELD IN PHYSICS TO PRELIMINARY ERROR ANALYSIS AND ERROR BUDGET

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and

Center for Space Research, The University of Texas at Austin, Austin, Texas 78712, USA

Received 3 May 1988
Revised 7 October 1988

The existence of the gravitomagnetic field, generated by mass currents according to Einstein geometrodynamics, has never been proved. The author of this paper, after a discussion of the importance of the gravitomagnetic field in physics, describes the experiment that he proposed in 1984 to measure this field using LAGEOS (Laser geodynamics satellite) together with another non-polar, laser-ranged satellite with the same orbital parameters as LAGEOS but a supplementary inclination.

The author then studies the main perturbations and measurement uncertainties that may affect the measurement of the Lense-Thirring drag. He concludes that, over the period of the node of ~ 3 years, the maximum error, using two nonpolar laser ranged satellites with supplementary inclinations, should not be larger than ~ 10% of the gravitomagnetic effect to be measured.

Table of Contents

IC IJMPA 1989: Analysis of the orbital perturbations affecting the nodes of LAGEOS-type satellites

(1) Use two LAGEOS satellites with supplementary inclinations

OR:

Use n satellites of LAGEOS-type to measure the first n-1 even zonal harmonics: J_2, J_4, … and the Lense-Thirring effect

3102 *Ignazio Ciufolini*

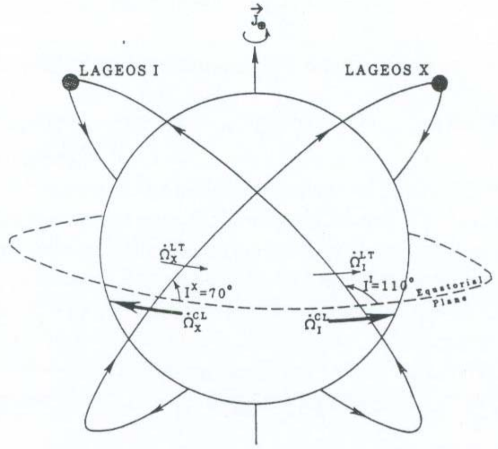

Fig. 5. The LAGEOS and LAGEOS X orbits and their classical and gravitomagnetic nodal precessions. A new[17] configuration to measure the Lense-Thirring effect.

For J_2, this corresponds, from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore, the uncertainty in $\dot{\Omega}^{\text{Class}}_{\text{Lageos}}$ is more than ten times larger than the Lense-Thirring precession.

A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (3.2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero. As mentioned before, Yilmaz proposed the use of polar satellites in 1959.[40,41] In 1976, Van Patten and Everitt[46,47] proposed an experiment with two drag-free, guided, counter-rotating, polar satellites to avoid inclination measurement errors.

A new solution[15,16,17,21,22,23] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS (see Fig. 5). Therefore, "LAGEOS X" should have the following orbital parameters:

$$I^X \cong \pi - I^I \cong 70°\,, \qquad a^X \cong a^I\,, \qquad e^X \cong e^I\,. \tag{3.3}$$

With this choice, since the classical precession $\dot{\Omega}^{\text{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\text{Class}}$ would be equal and opposite for the two satellites:

$$\dot{\Omega}^{\text{Class}}_X = -\dot{\Omega}^{\text{Class}}_I\,. \tag{3.4}$$

By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$ is independent of the inclination (Eq. (3.1)), $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites:

IL NUOVO CIMENTO Vol. 109 A, N. 12 Dicembre 1996

On a new method to measure the gravitomagnetic field using two orbiting satellites

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IFSI-CNR - Frascati, Italy
Dipartimento Aerospaziale, Università di Roma «La Sapienza» - Roma, Italy

(ricevuto il 20 Settembre 1996; approvato il 15 Novembre 1996)

Summary. — We describe a new method to obtain the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct detection of the gravitomagnetic field. This method is based on the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II. By this new approach one achieves a measurement of the gravitomagnetic field with accuracy of about 25%, or less, of the Lense-Thirring effect in general relativity.

PACS 11.90 – Other topics in general field and particle theory.
PACS 04.80.Cc – Experimental test of gravitational theories.

1. – The gravitomagnetic field, its invariant characterization and past attempts to measure it

Einstein's theory of general relativity [1, 2] predicts the occurrence of a «new» field generated by mass-energy currents, not present in classical Galilei-Newton mechanics. This field is called the gravitomagnetic field for its analogies with the magnetic field in electrodynamics.

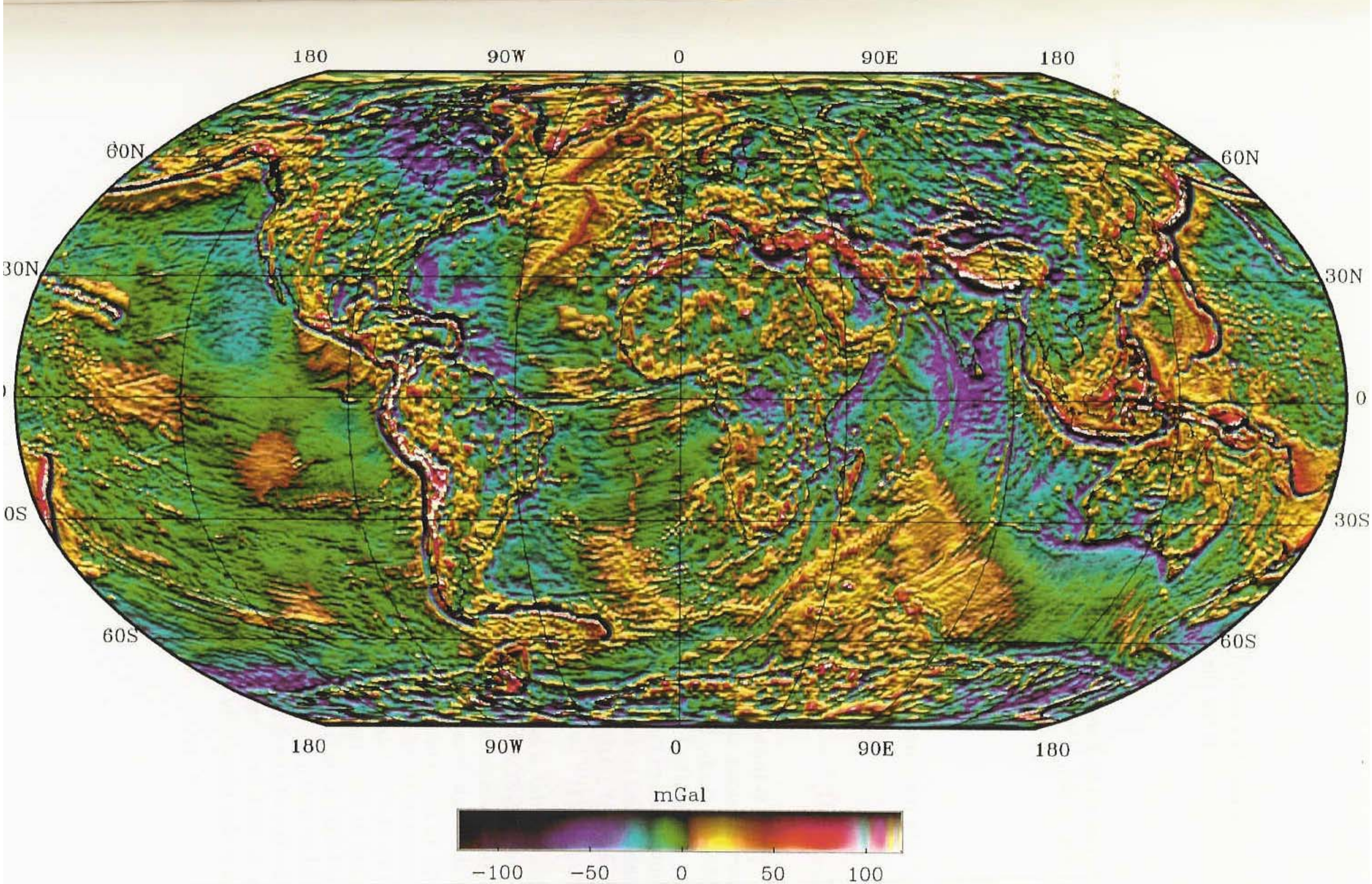
In general relativity, for a stationary mass-energy current distribution $\rho_m \boldsymbol{v}$, in the weak-field and slow-motion limit, one can write [2] the Einstein equation in the Lorentz gauge: $\Delta \boldsymbol{h} \cong 16\pi\rho_m \boldsymbol{v}$, where $\boldsymbol{h} \equiv (h_{01}, h_{02}, h_{03})$ are the $(0i)$-components of the metric tensor; $\boldsymbol{h}$ is called the gravitomagnetic potential. For a localized, stationary mass-energy distribution, in the weak-field and slow-motion limit, we can then write: $\boldsymbol{h} \cong -2((\boldsymbol{J} \times \boldsymbol{x})/r^3)$, where $\boldsymbol{J}$ is the angular momentum of the central body. In general relativity, one can also define [2] a gravitomagnetic field $\boldsymbol{H}$ given by $\boldsymbol{H} = \nabla \times \boldsymbol{h}$.

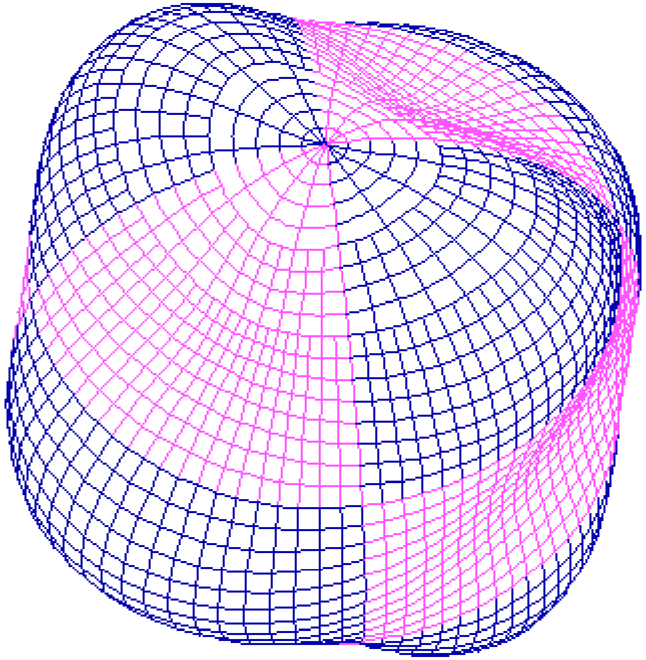
The Lense-Thirring effect is a consequence of the gravitomagnetic field and consists of a tiny perturbation of the orbital elements of a test particle due to the angular momentum of the central body. To characterize the gravitomagnetic field generated by the angular momentum of a body, and the Lense-Thirring effect, and distinguish it from other relativistic phenomena, such as the de Sitter effect, due to the

IC NCA 1996:
use the node of LAGEOS and the node of LAGEOS II to measure the Lense-Thirring effect

However, in 1996 the two nodes were not enough to measure the Lense-Thirring effect

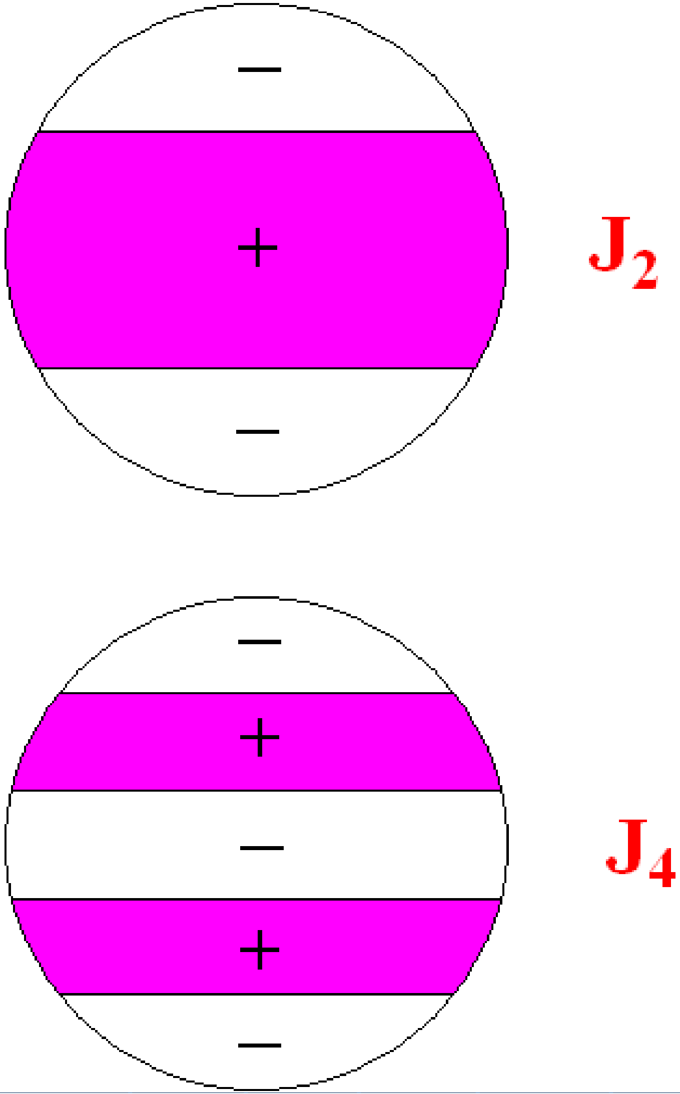
EGM-96 GRAVITY MODEL

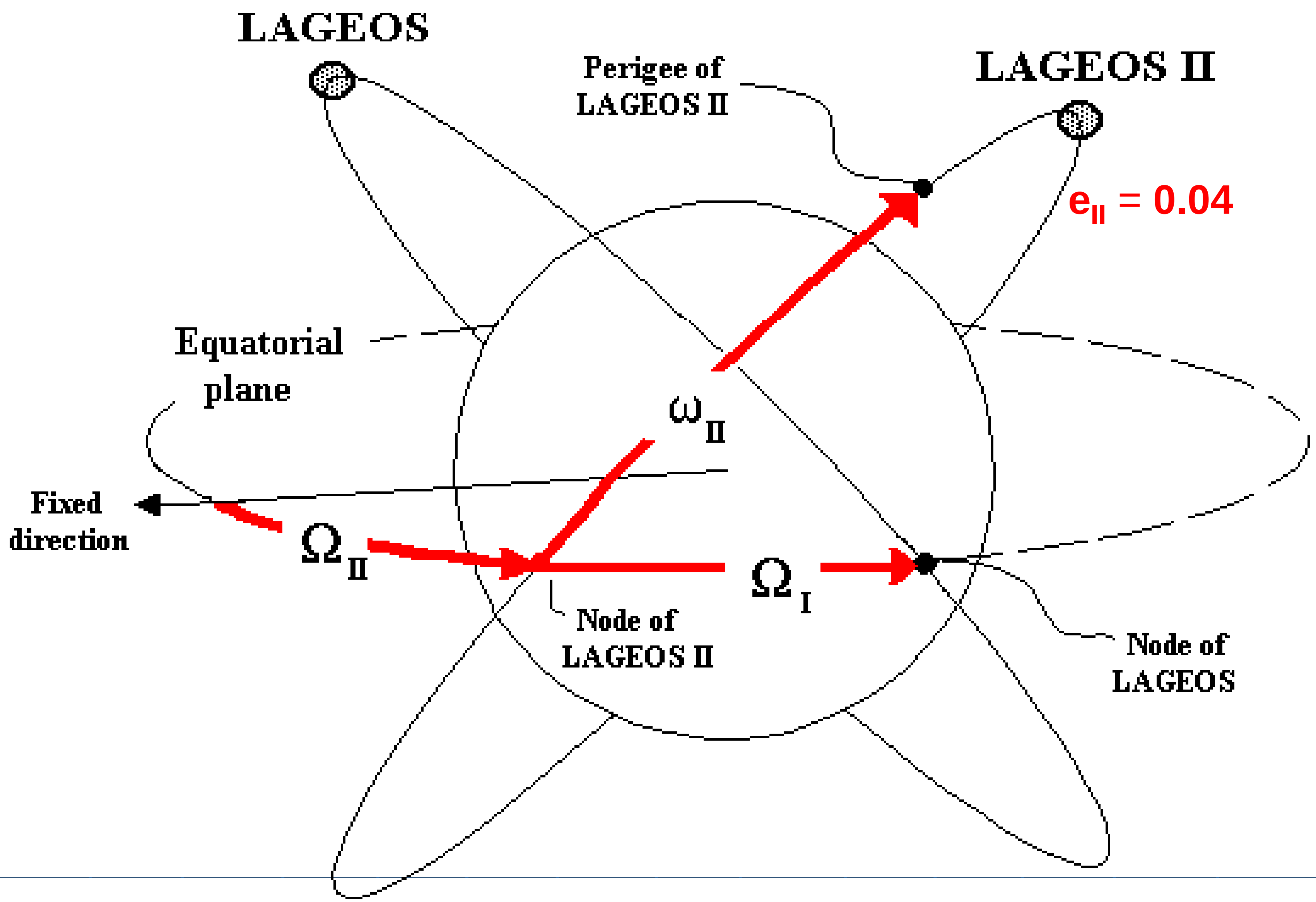




l=3, m=1

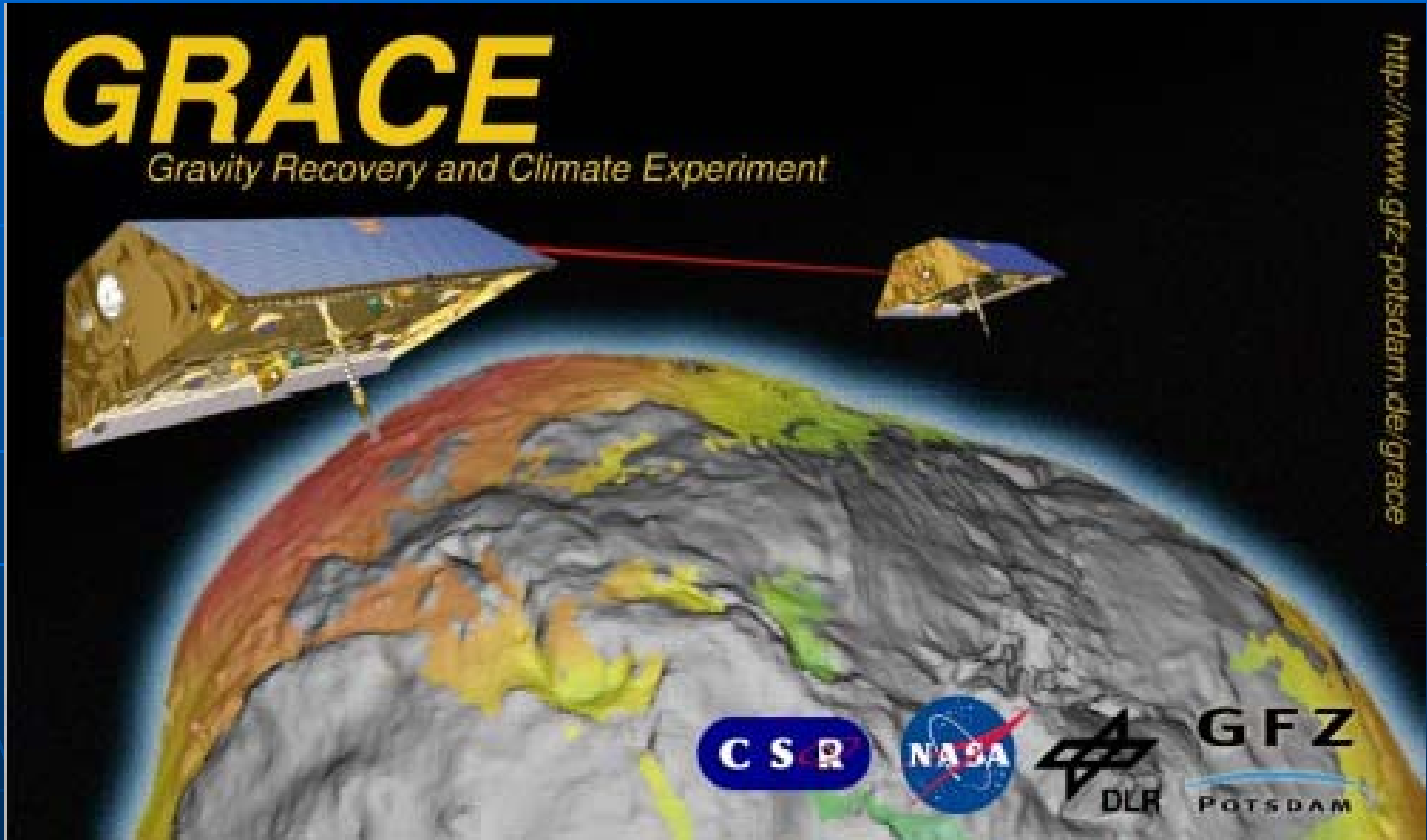
EVEN ZONAL HARMONICS




LAGEOS
Perigee of
LAGEOS II
LAGEOS II
$e_{II} = 0.04$
Equatorial
plane
ω_{II}
Fixed
direction
Ω_{II}
Ω_{I}
Node of
LAGEOS II
Node of
LAGEOS

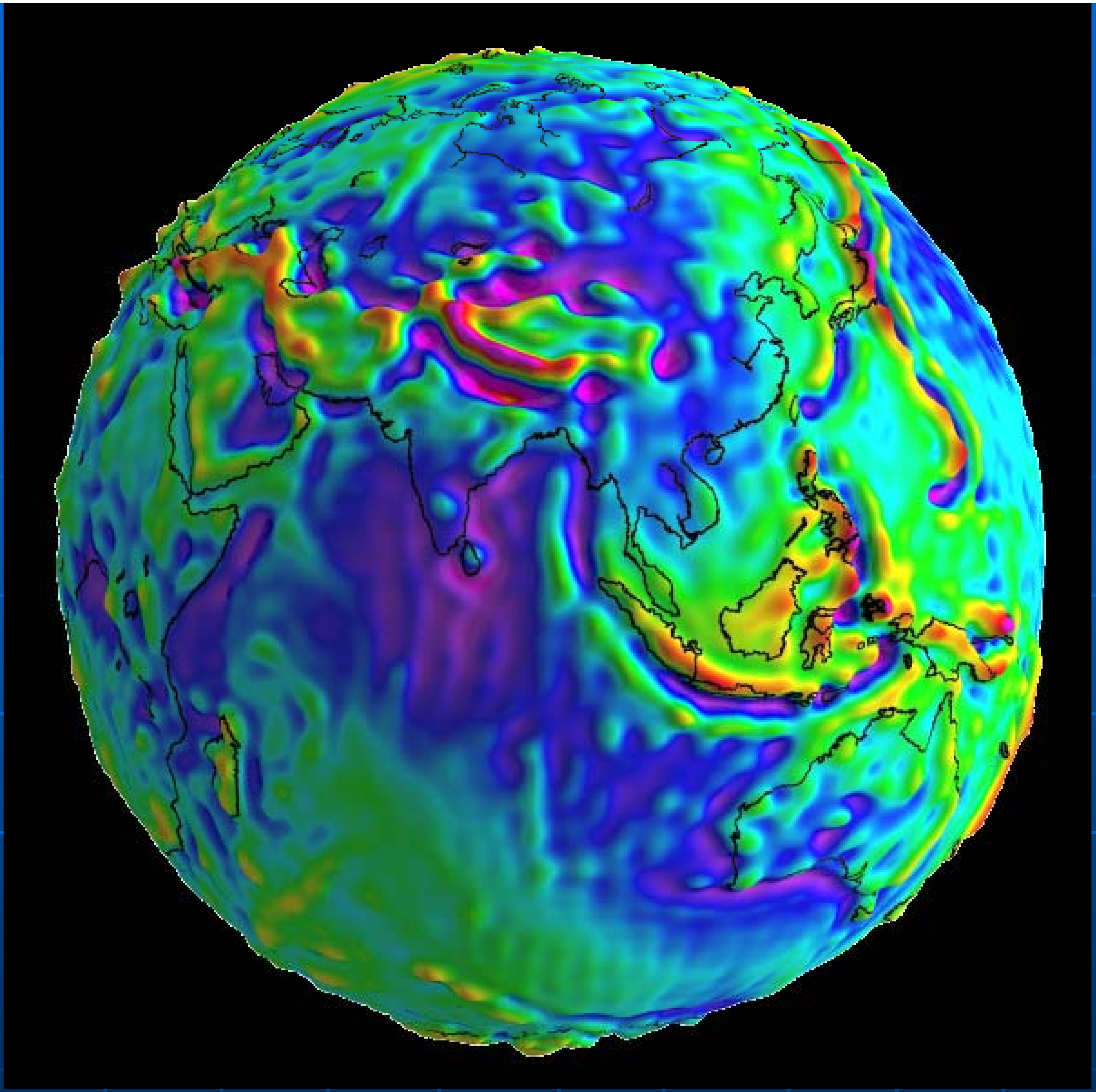
I.C., NC A, 1996

2002



Use of GRACE to test Lense-Thirring at a few percent level:
J. Ries et al. 2003 (1999),E. Pavlis 2002 (2000)


6 September 2007 | www.nature.com/nature | £10
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
nature
THE K/T IMPACT
Baptistina asteroids in the frame
BIOMETRICS
The questions you meant to ask
TSUNAMIS
Tracking risk off the Myanmar coast
THE RIDDLE OF INERTIA
How Earth's rotation reshapes space and time
NATUREJOBS
Hydrogen technology



EIGEN-GRACE-S (GFZ 2004)

EIGEN-GRACE02S Model and Uncertainties

Even zonals lm	Value $\cdot 10^{-6}$	Uncertainty	Uncertainty on node I	Uncertainty on node II	Uncertainty on perigee II
20	-484.16519788	$0.53 \cdot 10^{-10}$	$1.59\ \Omega_{LT}$	$2.86\ \Omega_{LT}$	$1.17\ \omega_{LT}$
40	0.53999294	$0.39 \cdot 10^{-11}$	$0.058\ \Omega_{LT}$	$0.02\ \Omega_{LT}$	$0.082\ \omega_{LT}$
60	-.14993038	$0.20 \cdot 10^{-11}$	$0.0076\ \Omega_{LT}$	$0.012\ \Omega_{LT}$	$0.0041\ \omega_{LT}$
80	0.04948789	$0.15 \cdot 10^{-11}$	$0.00045\ \Omega_{LT}$	$0.0021\ \Omega_{LT}$	$0.0051\ \omega_{LT}$
10,0	0.05332122	$0.21 \cdot 10^{-11}$	$0.00042\ \Omega_{LT}$	$0.00074\ \Omega_{LT}$	$0.0023\ \omega_{LT}$

Using EIGEN-GRACE02S:
2 main unknowns: δC_{20} and LT
Needed 2 observables:

$\delta\Omega_{I}$,$\delta\Omega_{II}$ (orbital angular momentum vector)

- $\delta\Omega_{I}$ = K_2 x δC_{20} + K_{2n} x $\delta C_{2n,0}$ + μ **(31 mas/yr)**
- $\delta\Omega_{II}$= K'_2 x δC_{20} + K'_{2n} x $\delta C_{2n,0}$ + μ **(31.5 mas/yr)**

$$\mu = \delta\Omega_{I} + K * \delta\Omega_{II}:$$

not dependent on δC_{20}

free from non-gravitational errors on the perigee

TOTAL ERROR FROM EVEN ZONALS ρ C40 =
= 3% to 4 % Lense-Thirring

I.C. PRL 1986; I.C. IJMP A 1989;
I.C. NC A, 1996; I.C. Proc. I SIGRAV School, Frascati 2002, IOP.

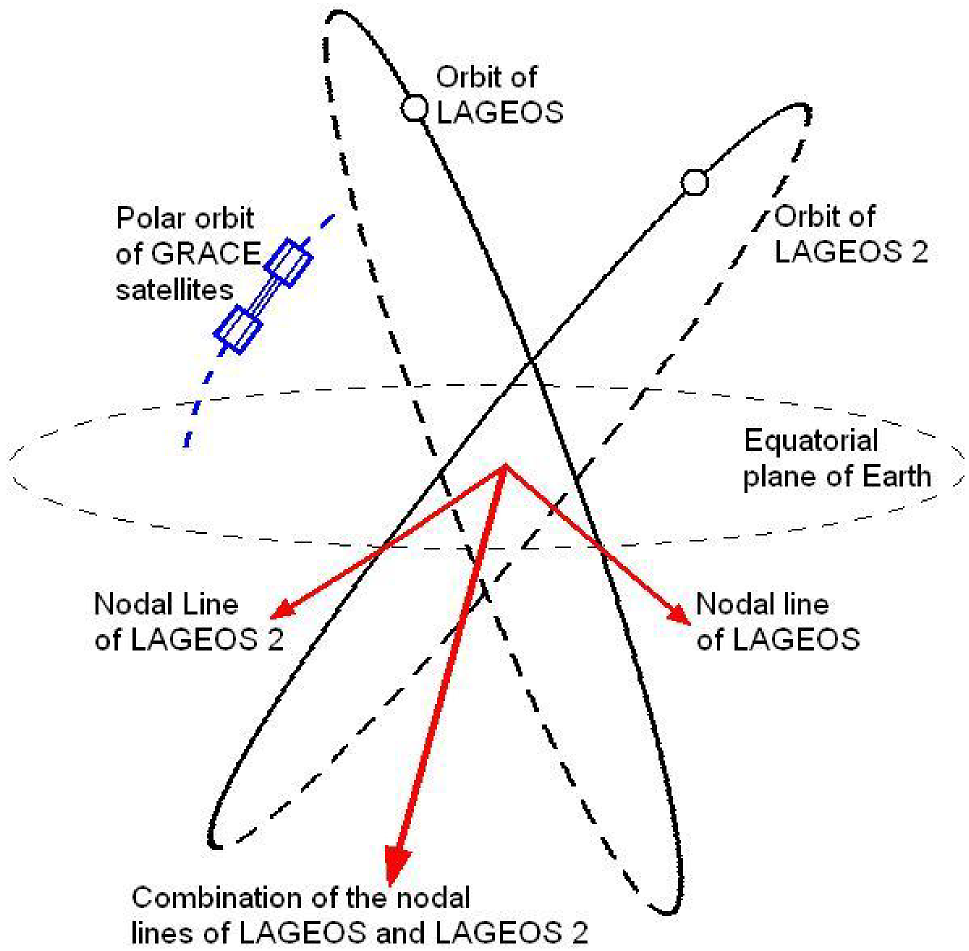
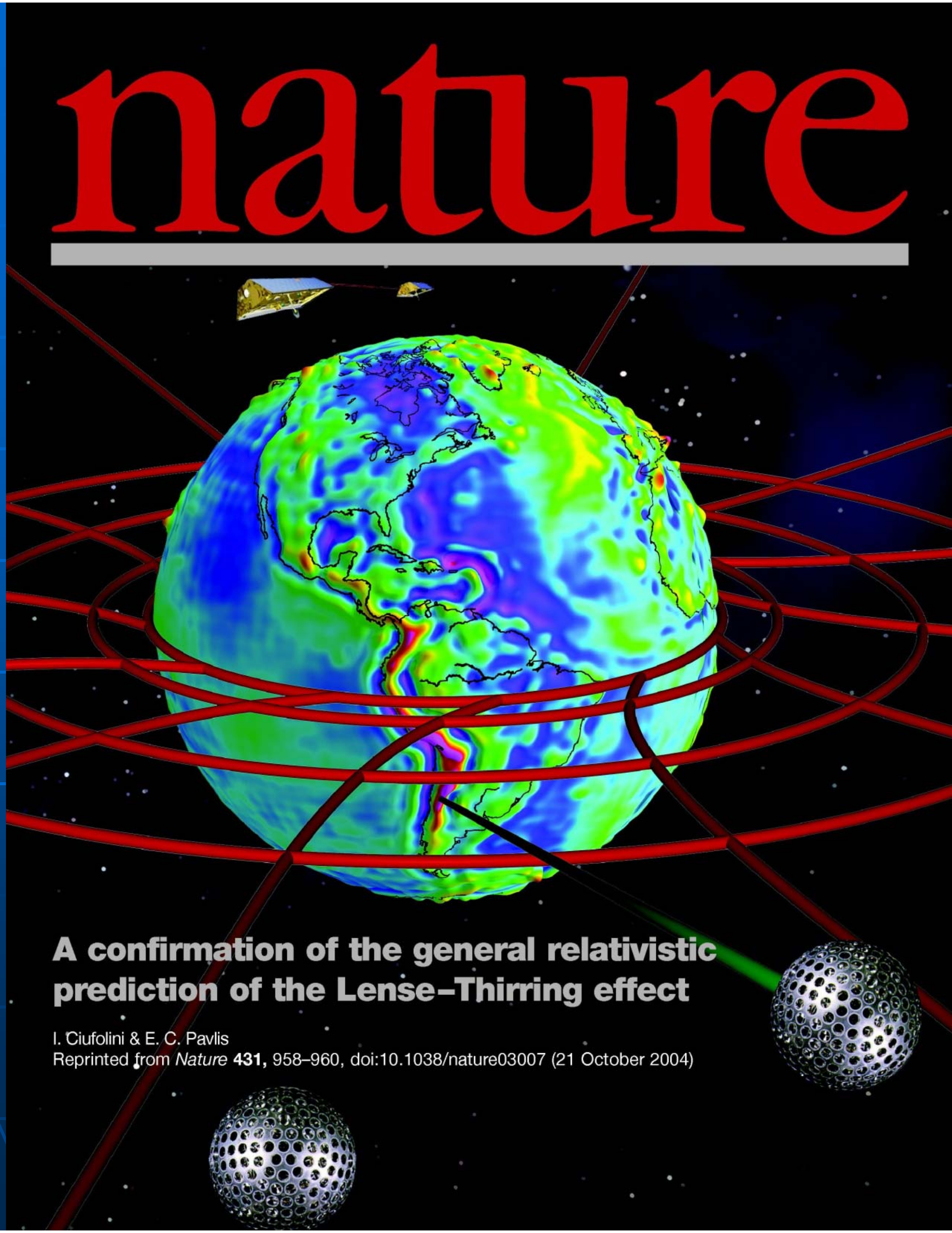
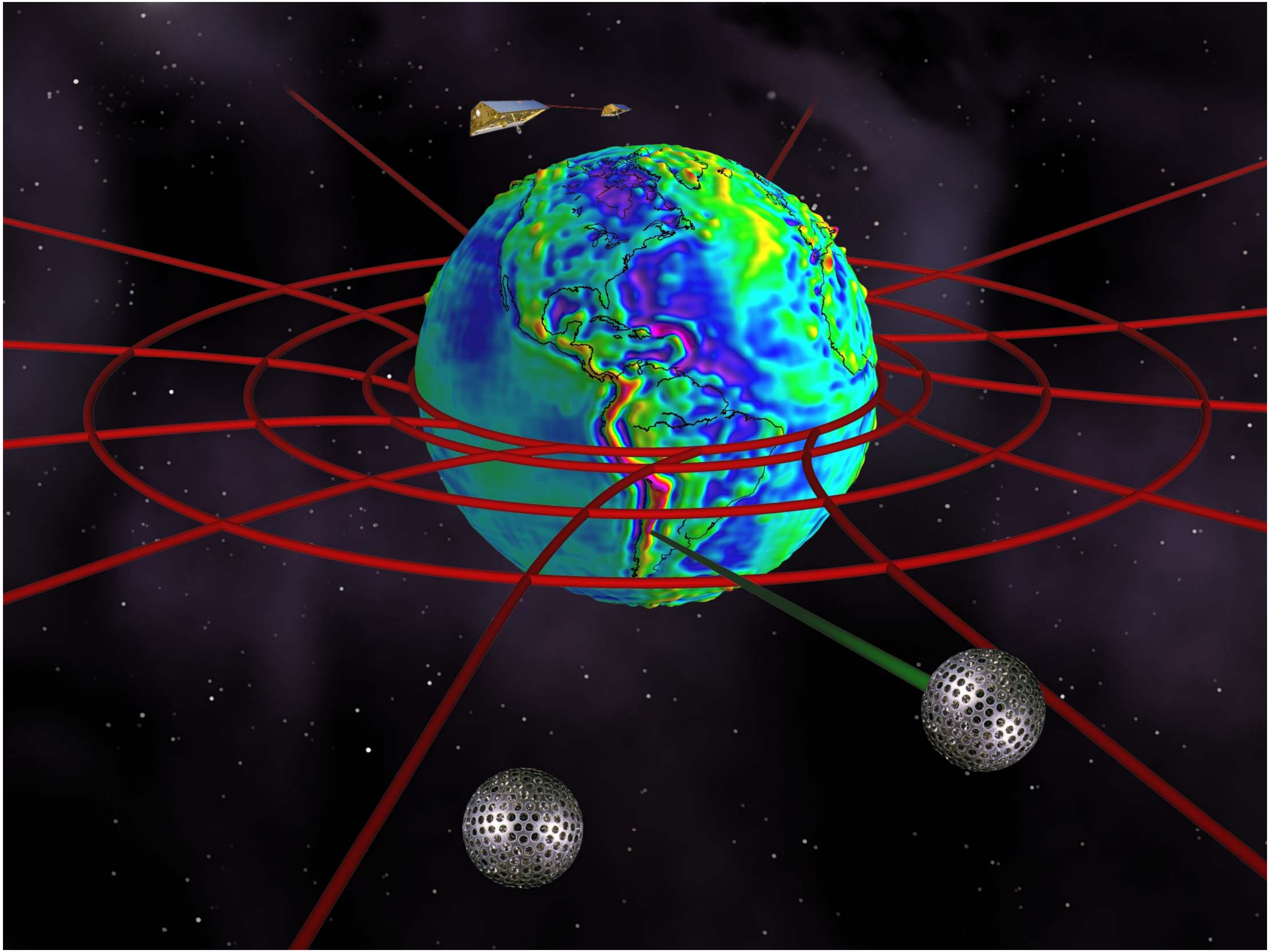

Orbit of
LAGEOS
Polar orbit
of GRACE
satellites
Orbit of
LAGEOS 2
Equatorial
plane of Earth
Nodal Line
of LAGEOS 2
Nodal line
of LAGEOS
Combination of the nodal
lines of LAGEOS and LAGEOS 2

Figure 1


nature
A confirmation of the general relativistic prediction of the Lense–Thirring effect
I. Ciufolini & E. C. Pavlis
Reprinted from *Nature* **431,** 958–960, doi:10.1038/nature03007 (21 October 2004)



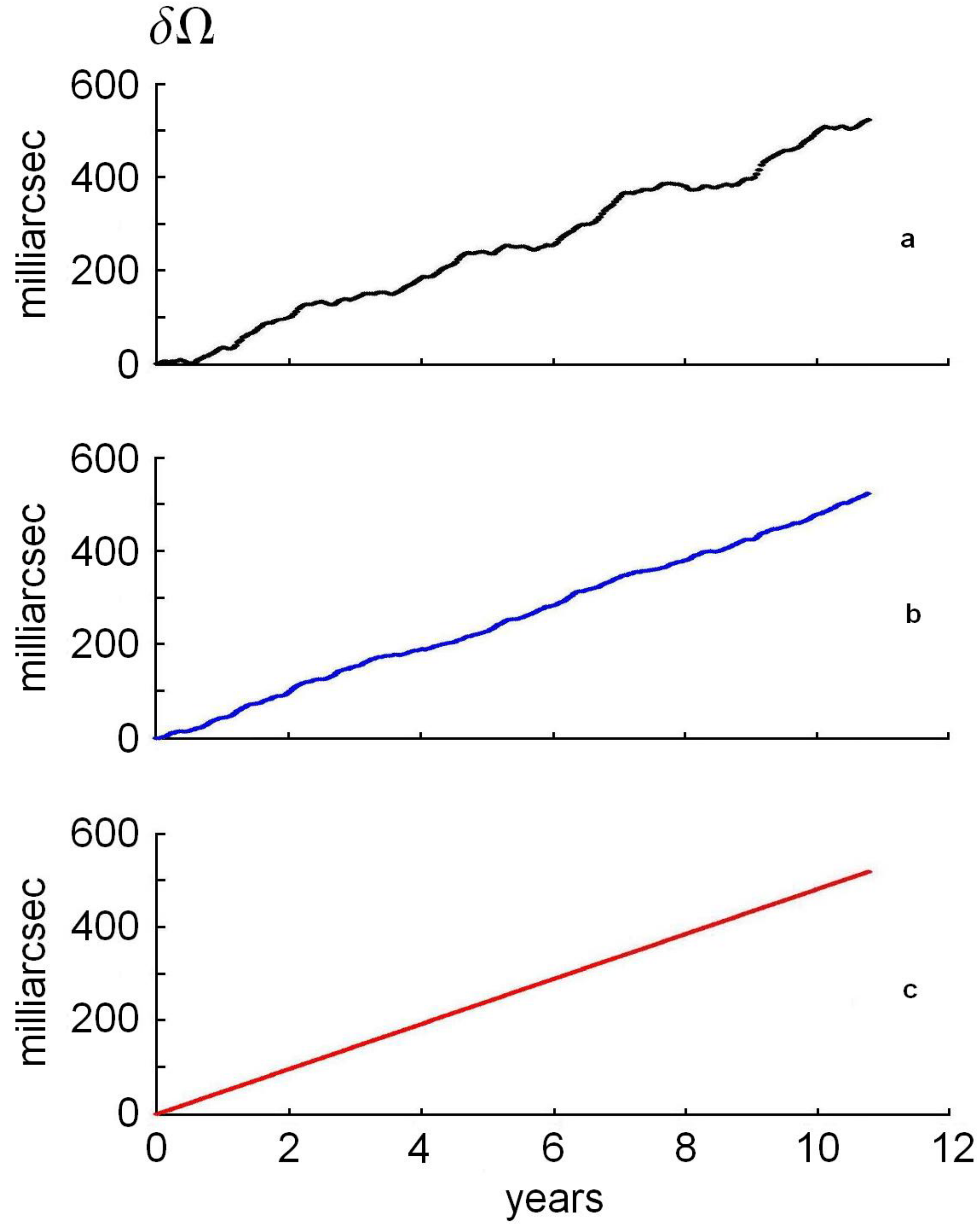


Figure 2

Observed value of
Lense-Thirring effect using
The combination of the
LAGEOS nodes.

Observed value of
Lense-Thirring effect = 99%
of the general relativistic
prediction. Fit of linear trend
plus 6 known frequencies

General relativistic
Prediction = 48.2 mas/yr

I.C. & E.Pavlis,
Letters to NATURE,
431, 958, 2004.

Error budget

- **Static gravitational field (using the EIGEN-GRACE02S uncertainties):**

  3 % to 4 % (the EIGEN-GRACE02S uncertainties include systematic errors) or 6 % to 8 % doubling the uncertainty published with EIGEN-GRACE02S (or 9 % to 12 % by tripling these uncertainties) .

  Time dependent gravitational field error:

  2 %

- **Non-Gravitational perturbations:**

  2 % to 3% [most of the modeling errors due to the non-gravitational perturbations are on the perigee, in particular due the Yarkowski effect on the perigee, but with in this combination we only used the nodes]

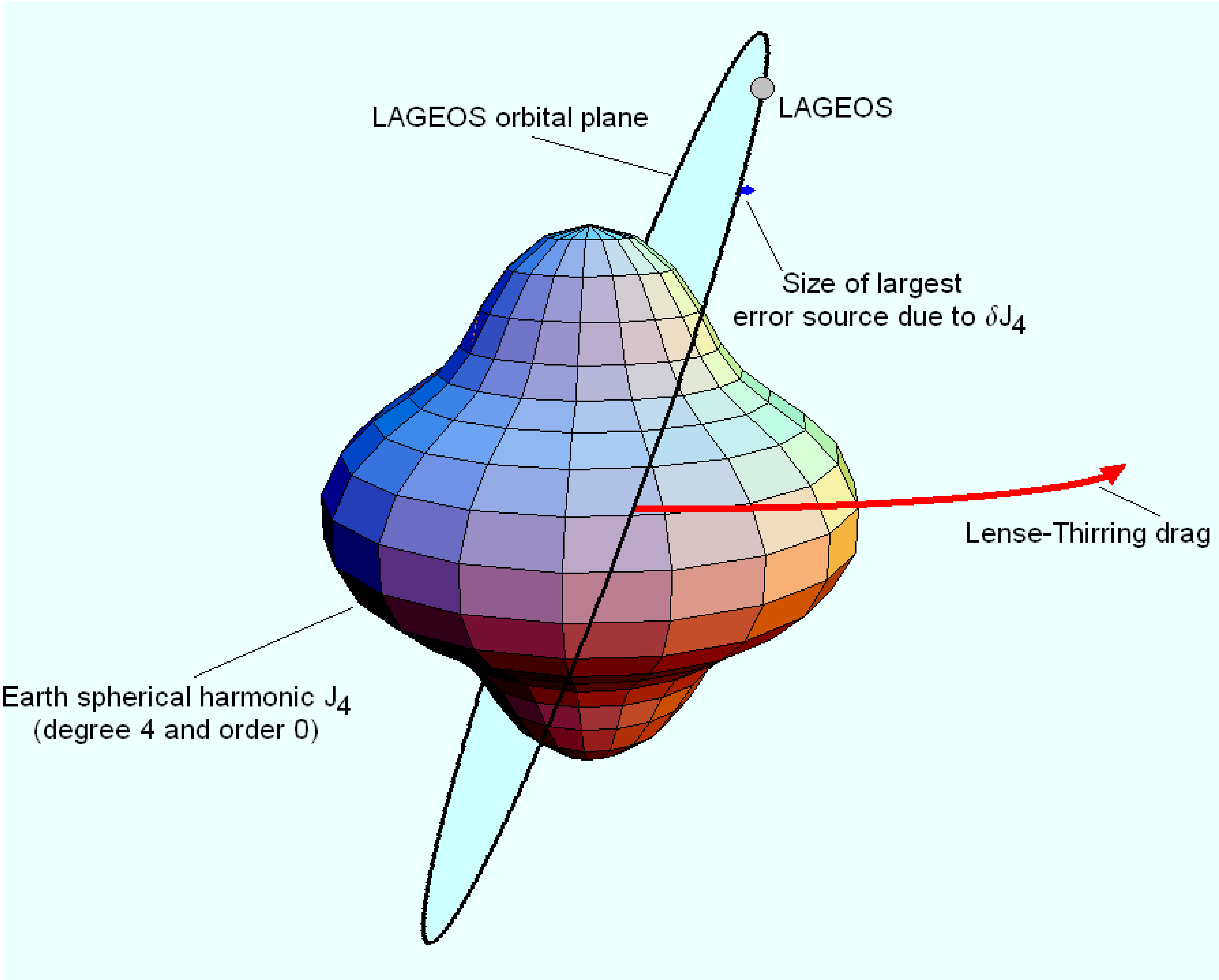
- 2% error due to random and stochastic errors and other errors

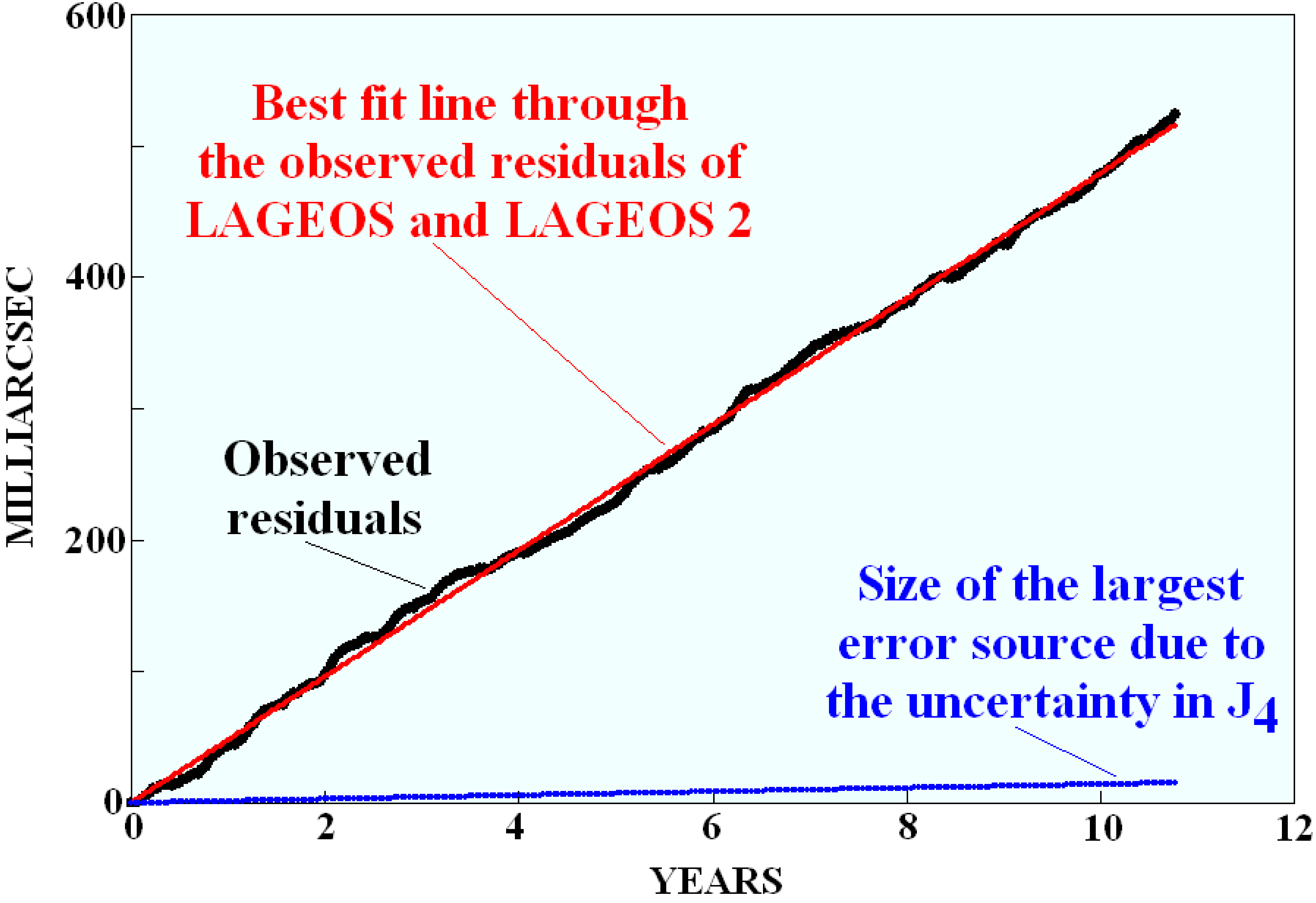
TOTAL: about 10 % (RSS)

I.C., E. Pavlis and R. Peron, New Astronomy (2006).

I.C. and E. Pavlis, New Astronomy (2005).

I.C. et al., in “General Relativity and John Wheeler”, Springer


LAGEOS orbital plane
LAGEOS
Size of largest
error source due to δJ_4
Lense-Thirring drag
Earth spherical harmonic J_4
(degree 4 and order 0)


Best fit line through
the observed residuals of
LAGEOS and LAGEOS 2
Observed
residuals
Size of the largest
error source due to
the uncertainty in J_4
MILLIARCSEC
YEARS
600
400
200
0
0
2
4
6
8
10
12

The 2004 analysis with EIGENGRACE02S:

- Does not use the perigee (i.e., no problems to assess the non-gravitational errors)
- In the error analysis we have summed up the absolute values of the errors due to each individual even zonal harmonic uncertainty: thus we did not use the correlation (anyhow small) among the even zonal harmonic coefficients
- The EIGENGRACE02S model was obtained with the use of GRACE data only and did NOT use any LAGEOS data
- The even zonal harmonics obtained from GRACE are independent of the Lense-Thirring effect (the acceleration of a polar, circular orbit satellite generated by the even zonals is orthogonal to the acceleration generated by the Lense-Thirring effect).

Potentially weak points of the 2004 analysis:

•The analysis was performed with the NASA orbital Estimator GEODYN, but what would happen by Performing it with a different orbital estimator ?

•The 2004 analysis was perfomed with EIGENGRACE02S but what happens if we change the gravity field model (and the corresponding value of the even zonal harmonics) ?

Answer:

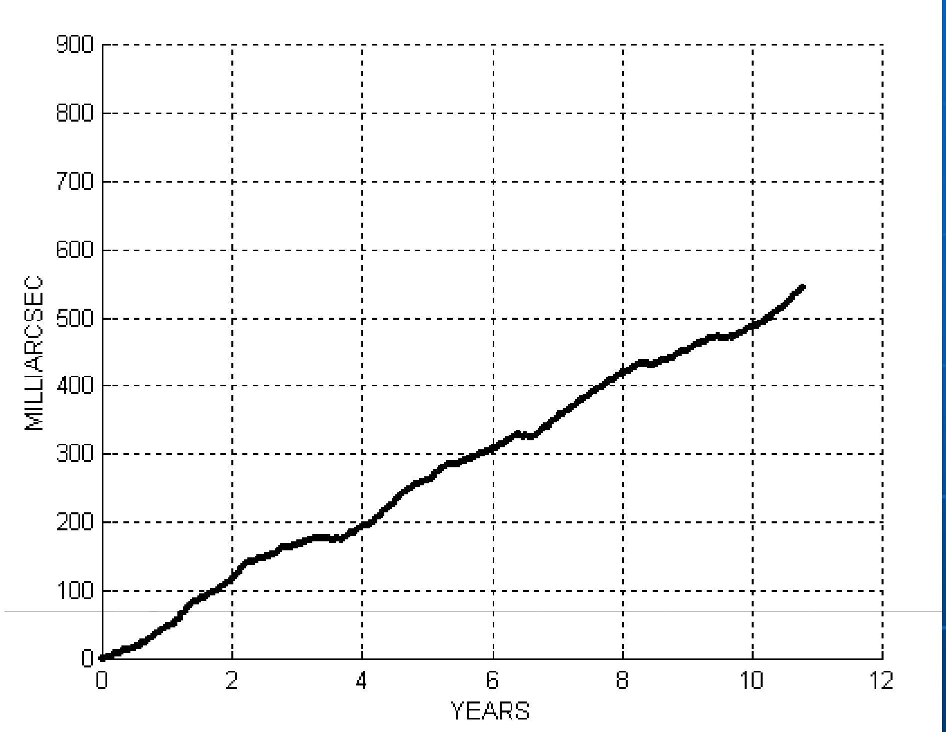
•Let us use the GFZ German orbital estimator EPOS (independent of GEODYN)

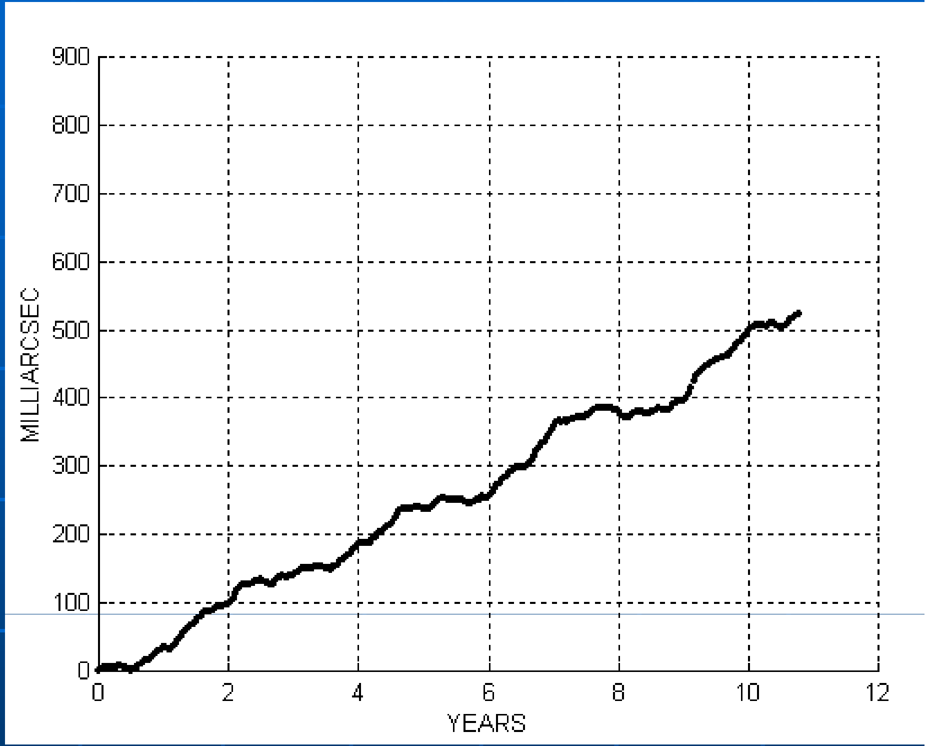
•Let us use different gravity field models obtained using GRACE

IC (Univ. Lecce), E. Pavlis (Univ Maryland Baltimore County),
R. Koenig and Neumayer (GFZ Munich/Potsdam),
G. Sindoni and A. Paolozzi (Univ. Roma I),
R. Matzner (Univ. Texas, Austin)

Using GEODYN (NASA) and EPOS (GFZ)

NEW 2006-2007 ANALYSIS OF THE LAGEOS ORBITS USING THE GFZ ORBITAL ESTIMATOR **EPOS**

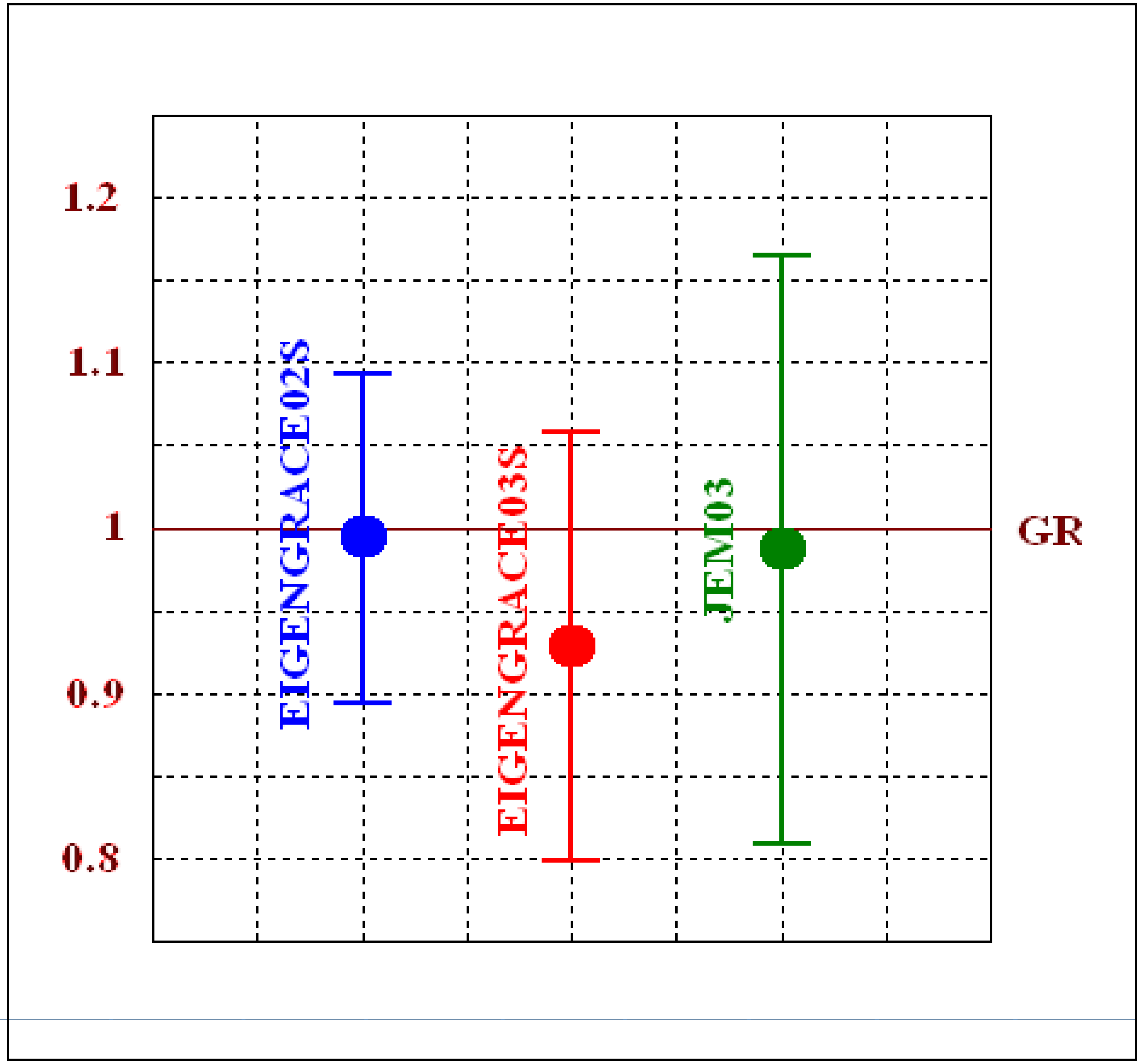


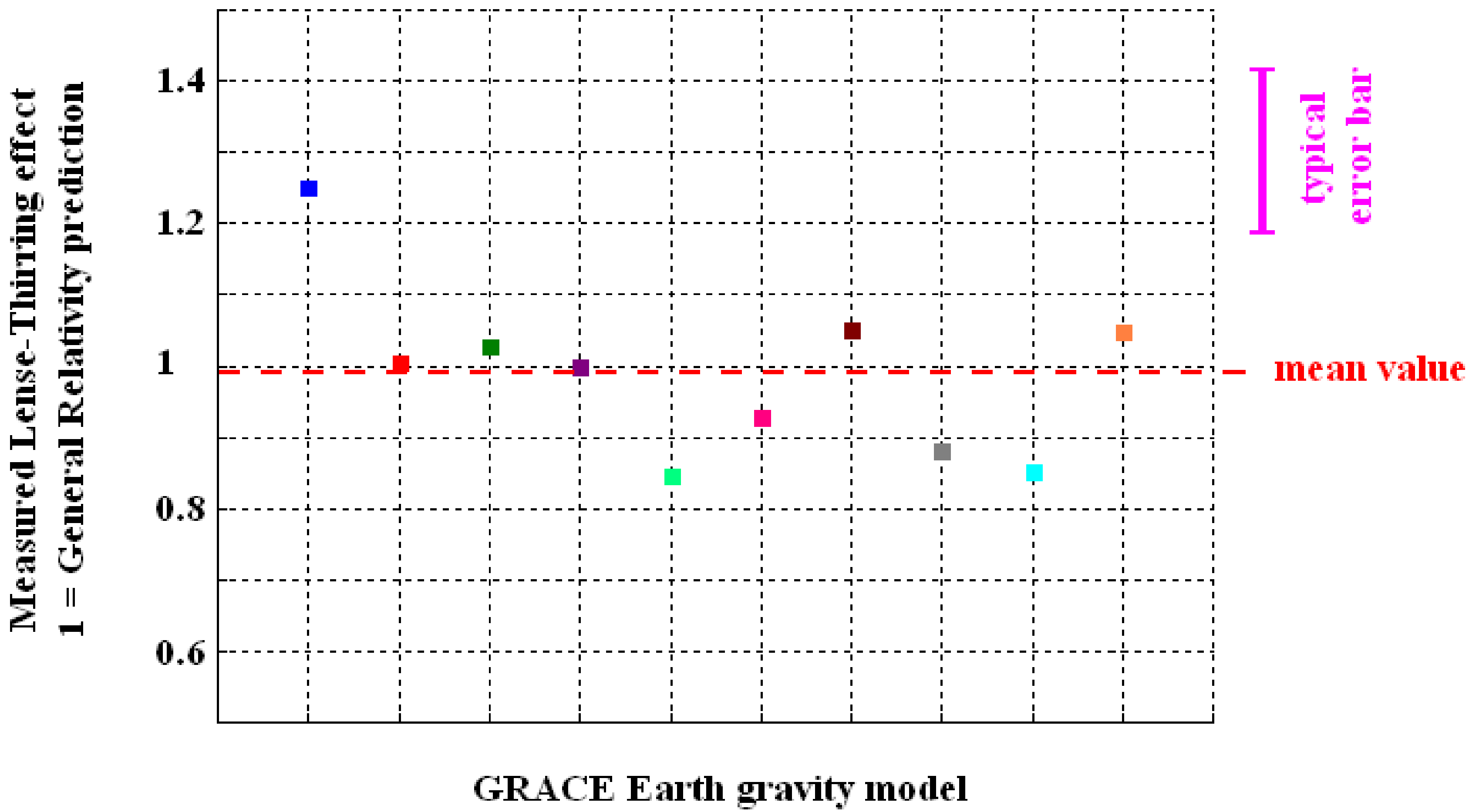


*by adding the geodetic precession of the orbital plane of an Earth satellite in the EPOS orbital estimator.

OLD 2004 ANALYSIS OF THE LAGEOS ORBITS USING THE NASA ORBITAL ESTIMATOR **GEODYN**

Comparison of Lense-Thirring effect measured using different Earth gravity field models

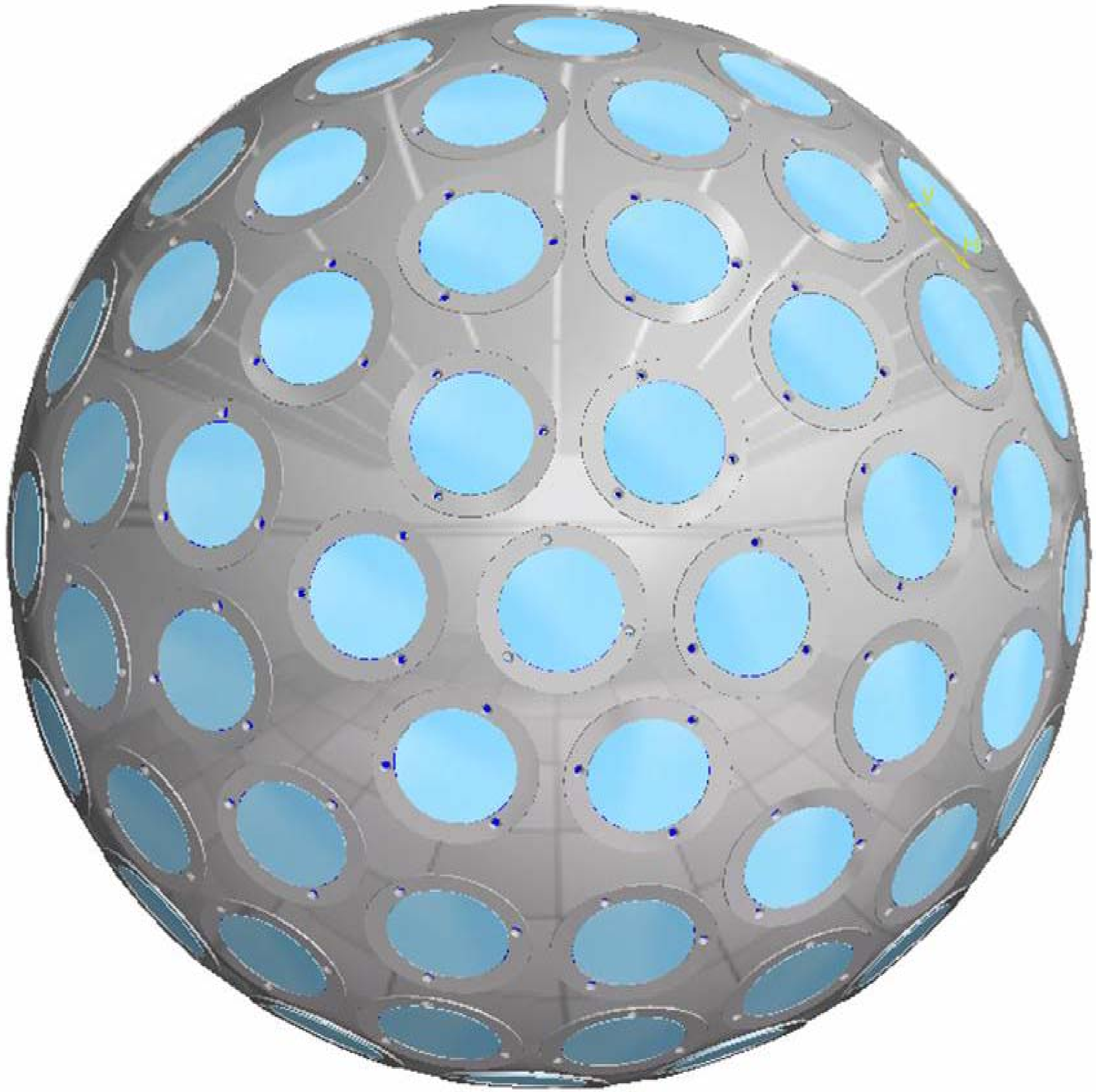




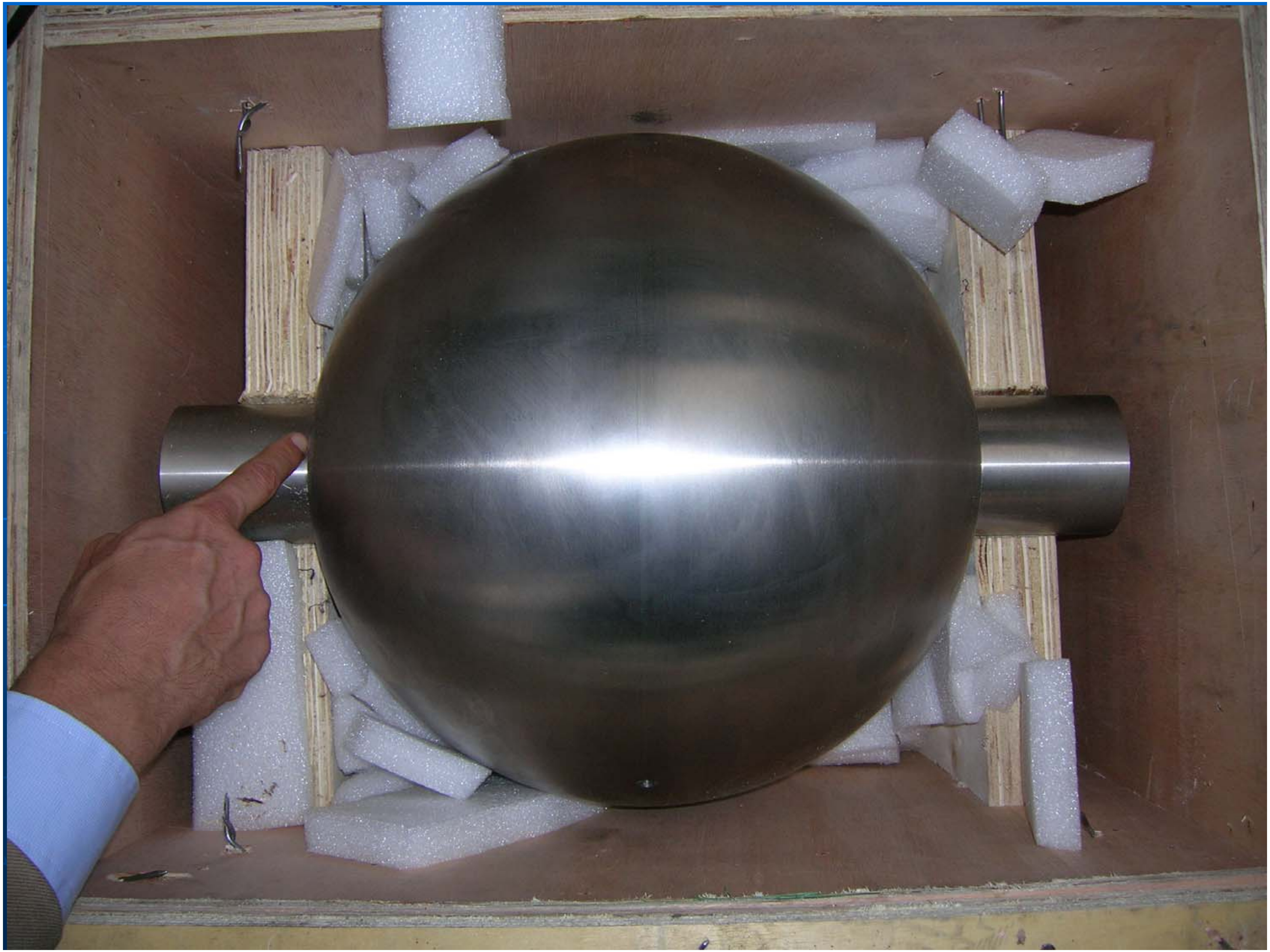
Ries et al. independent results for the measurement of frame.draggging by spin using LAGEOS, LAGEOS 2 and the GRACE Earth's gravity models.
John Ries (UT Austin) error budget about 12 %.

LARES
(LAser RElativity Satellite)

- Weight about 400 kg
- Radius about 18 cm
- Material Solid sphere of Tungsten alloy
- Semimajor Axis about 7900 km
- Eccentricity nearly zero
- Inclination about 71.5 degrees
- Combined with LAGEOS and LAGEOS 2 data it would provide a confirmation of Einstein General Relativity, the measurement of frame-dragging with accuracy of a few percent.




esa
vega
esa
VEGA



ERROR ANALYSIS FOR THE LARES EXPERIMENT AND FOR THE LAGEOS-2004 RESULTS

ERRORS IN THE LARES AND LAGEOS MESUREMENTS OF THE LENSE-THIRRING EFFECT MAY BE DIVIDED IN GRAVITATIONAL, NON-GRAVITATIONAL and MEASUREMENT ERRORS

- **THE MAIN-NON GRAVITATIONAL SOURCES OF ERRORS ARE THE UNCERTAINTIES IN THE MODELLING OF RADIATION PRESSURE FROM SUN AND FROM EARTH (ALBEDO), THERMAL THRUST EFFECTS (YASKOVSKI AND RUBINCAM EFFECTS), PARTICLE DRAG**
- **THE MAIN GRAVITATIONAL SOURCES OF ERRORS ARE THE UNCERTAINTIES IN THE MODELLING OF THE NEWTONIAN GRAVITATIONAL FIELD OF EARTH, BOTH STATIC AND TIME-DEPENDENT (HOWEVER WE ALSO INCLUDE IN OUR ANALYSIS THE GRAVITATIONAL FIELD OF MOON AND PLANETS), AMONG THESE, BY FAR THE MAIN SOURCE OF ERROR IS DUE TO THE AXIALLY SYMMETRIC PART OF THE EARTH GRAVITATIONAL FIELD AND TO THE EVEN ZONAL HARMONICS (EVEN DEGREE AND ZERO ORDER) OF THE SPHERICAL HARMONIC EXPANSION OF THE EARTH POTENTIAL.**

Equation describing the classical rate of change of the node of a satellite as a function of its orbital parameters, a,I, e, and Earth's parameters: mass, radius and even zonal harmonics J2, J4, …

$$\dot{\Omega}_{Class} = -\frac{3}{2}\, n\, \frac{\cos I}{(1-e^2)^2} \left\{ J_2 \left(\frac{R_\oplus}{a}\right)^2 + J_4 \left(\frac{R_\oplus}{a}\right)^4 \left[\frac{5}{8}\,(7\ \sin^2 I - 4)\, \frac{(1+\frac{3}{2}e^2)}{(1+e^2)^2}\right]\right\}$$

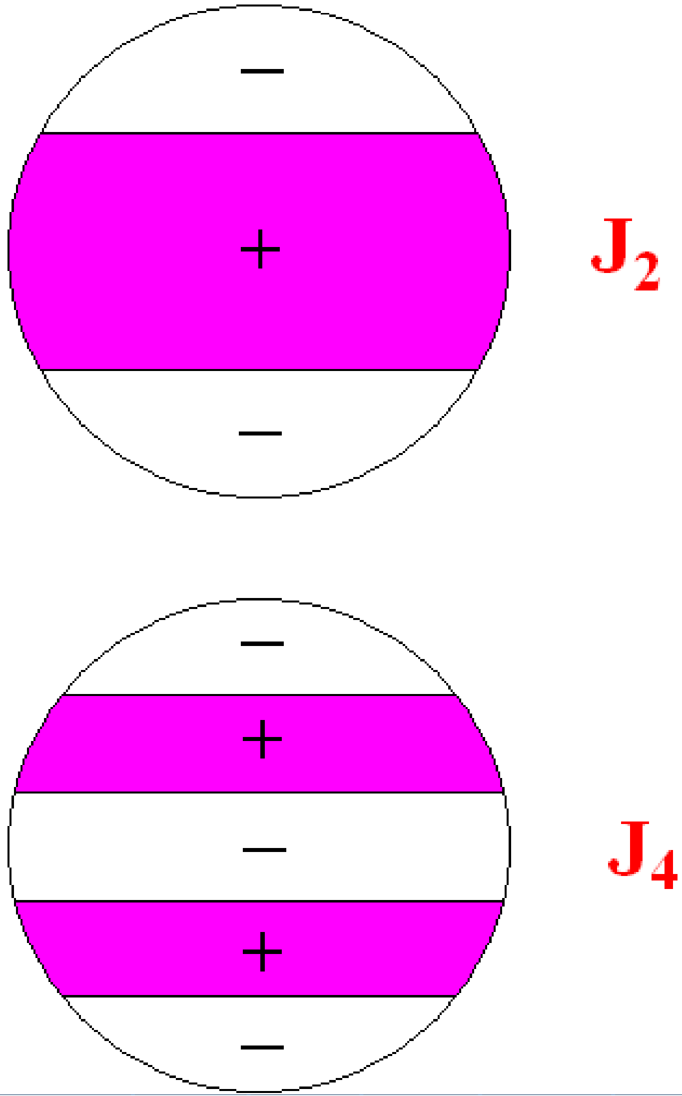
In order to measure the Lense-Thirring effect this classical node precession must be accurately enough modeled (i.e., its behavior must be predicted on the basis of the available physical models), i.e., it must be modeled at the level of a milliarcsec compared to the Lense-Thirring effect (of size of about 31 milliarcsec

Every quantity in this equation can be determined accurately enough via satellite laser ranging to LAGEOS, LAGEOS 2 and LARES for a 1 % measurement of the Lense-Thirring effect, apart from the even zonal harmonics J2, J4, … ,

GRAVITATIONAL ERRORS

Even zonal harmonics, of degree even and zero order, are the axially symmetric deviations of the Earth potential (of even Degree from) spherical symmetry.

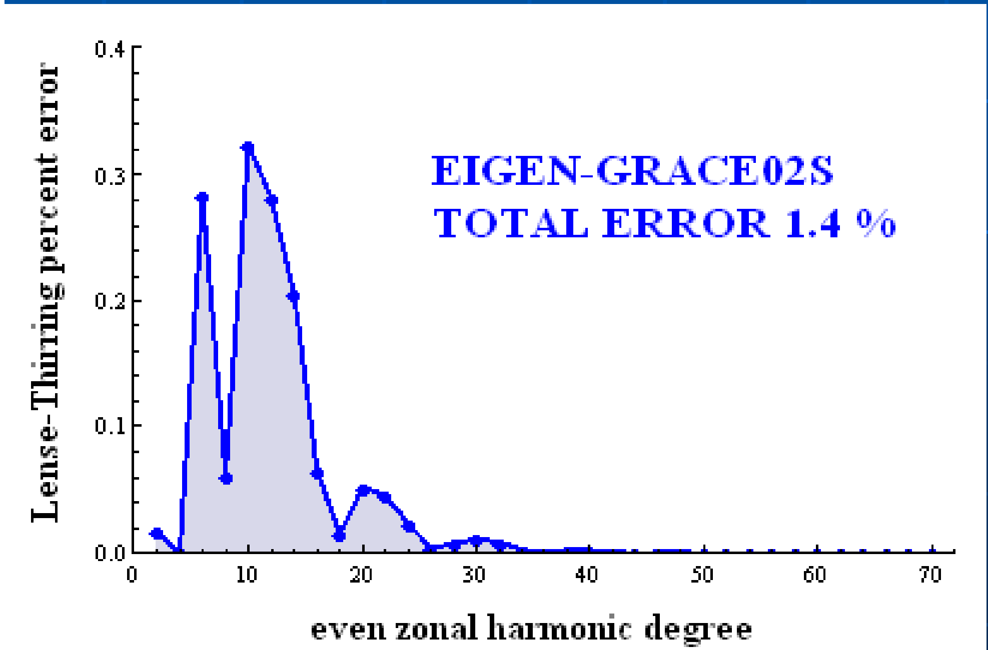
EVEN ZONAL HARMONICS



GRAVITATIONAL ERRORS

- Using the Earth gravitational model EIGEN-GRACE02S (February 2004), based on 111 days of GRACE observations, i.e., propagating the uncertainties of EIGEN-GRACE02S published by GFZ Potsdam on the nodes of LAGEOS, LAGEOS 2 and LARES and their combination, we find a total error of 1.4 %.

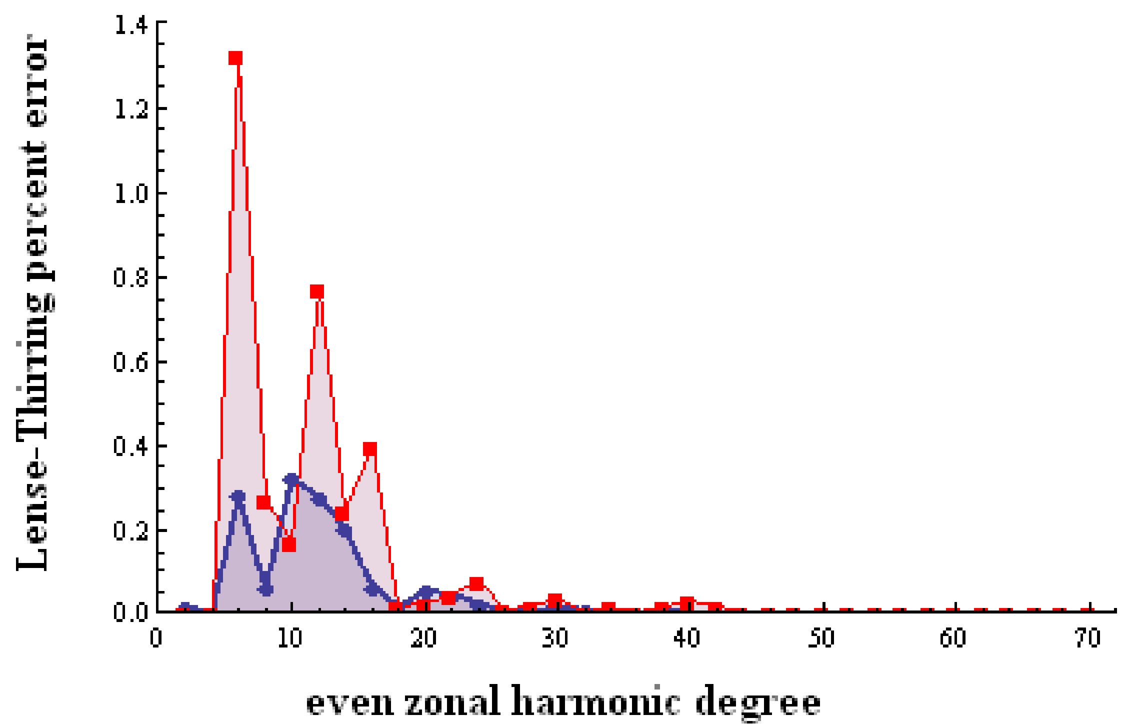
In particular we have calculated the error induced by the uncertainty of each even zonal harmonic up to degree 70: after degree 26 the error is negligible.



By the time of the LARES data analysis (2012-2015) we can assume an improvement in the GRACE Earth gravity field models of about one order of magnitude, thanks to much longer GRACE observations with respect to 110 days of EIGEN-GRACE02S and also to GOCE (2008).

GRAVITATIONAL ERRORS

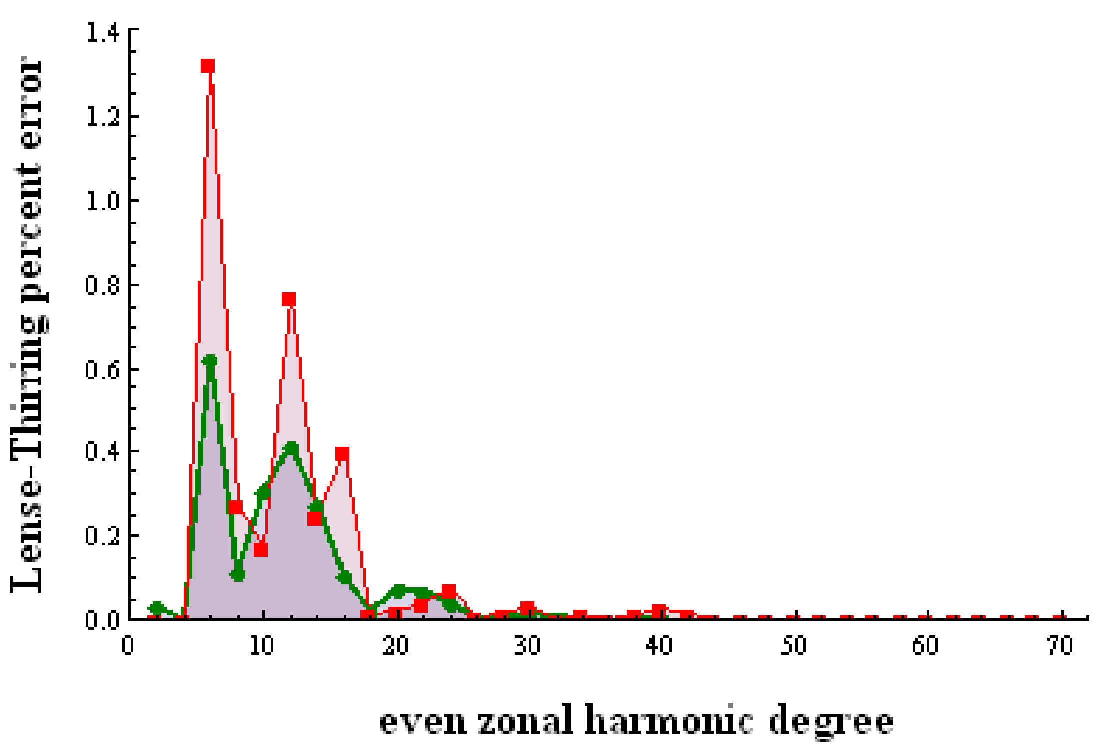
Standard technique in space geodesy to estimate the reliability of the published uncertainties of an Earth gravity model: take the difference between each harmonic coefficient of that model with the same harmonic coefficient of a differnt model and compare this difference with the published uncertainties. Let us take difference between each harmonic of the EIGEN-GRACE02S (GFZ Potsdam) model minus the same harmonic in the GGM02S (CSR Austin) model. CAVEAT: in order to use this technique, one must difference models of comparable accuracy, i.e., models that are indeed comparable, or use this method to only evaluate the less accurate model!



In Blue: percent errors in the measurement of the Lense-Thirring effect for EIGEN-GRACE02S for each even zonal

In Red: percent errors in the measurement of the Lense-Thirring effect using the difference between EIGEN-GRACE02S and GGM02S for each even zonal

GRAVITATIONAL ERRORS



In Green: percent errors in the measurement of the Lense-Thirring effect for GGM02S for each even Zonal harmonic

In Red: percent errors in the measurement of the Lense-Thirring effect Using the difference between EIGEN-GRACE02S and GGM02S for each even zonal harmonic

MEASUREMENT ERRORS

The main measurement error is due to the bias in the measurement of the inclination due to atmospheric refraction mismodelling.

The atmospheric refraction mismodeling on LAGEOS and LAGEOS 2, by comparing different models and different methods of observations has been estimated to be 0.030 milliarcsec, average, for LAGEOS and 0.010 milliarcsec, average, for LAGEOS 2 (see Mendes and Pavlis, Geophysical Res. Lett., 31, 2004).

From the LAGEOS and LAGEOS 2 nodal rate equations we have that these uncertainties in the inclination produce an mismodeled nodal rate corresponding to: 0.6 % of the Lense-Thirring effect for LAGEOS and 0.36 % for LAGEOS 2 and of about 0.5 % in the combination of their nodes.

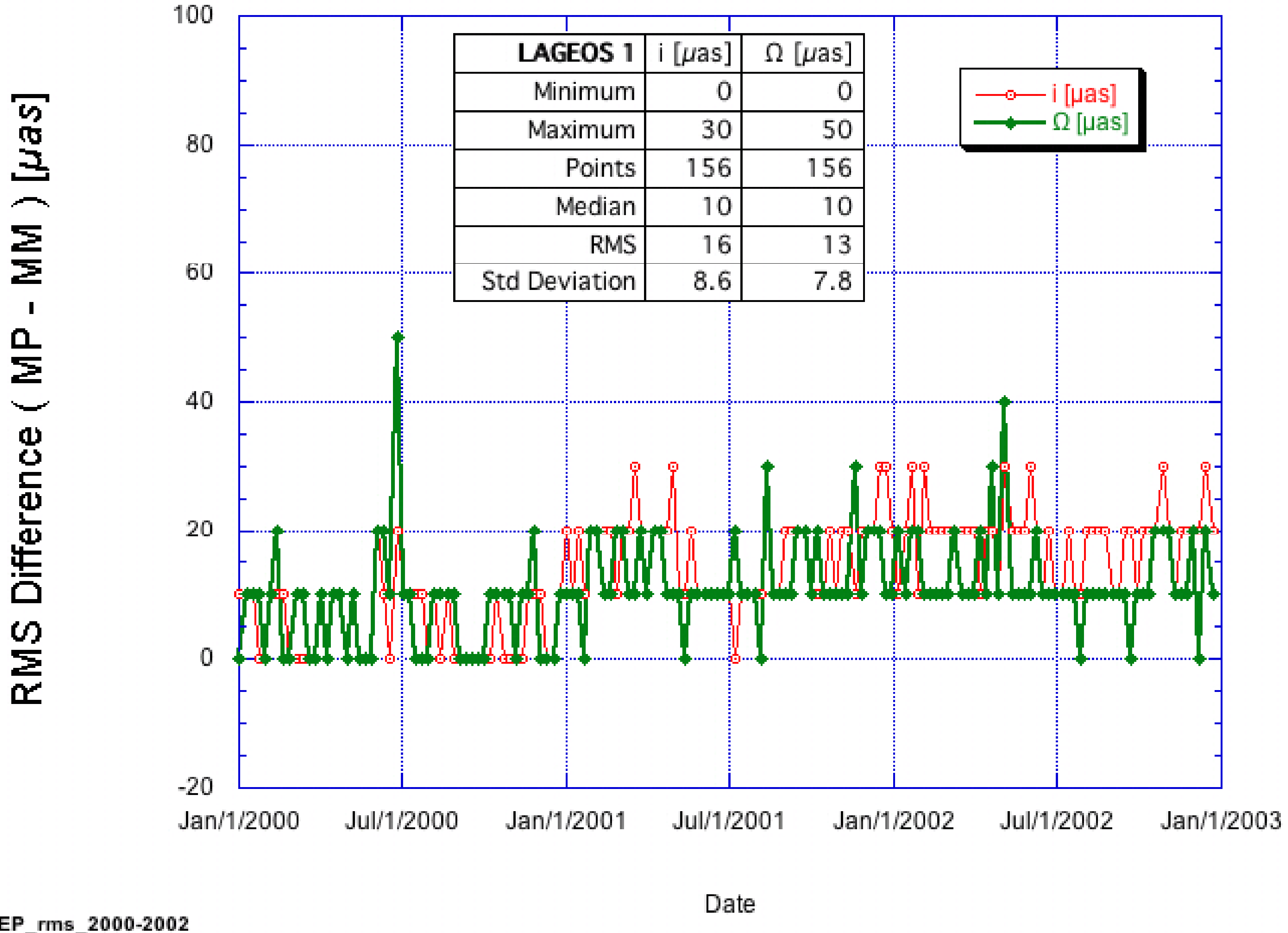
Differences in the inclinations of LAGEOS and LAGEOS 2 between Marini-Murray (MM) and the new Mendes-Pavlis (MP) model:
the differences between the two atmospheric refraction models are for the inclination at the level of 0.01 milliarcseconds for LAGEOS 2 and 0.03 milliarcseconds for LAGEOS 2

THE ERROR DUE TO NON-GRAVITATIONAL PERTURBATIONS IS LESS THAN 1 %, IN PARTICULAR:

•RADIATION PRESSURE FROM SUN AND EARTH CAN BE MODELLED WITH HIGH ACCURACY SINCE THE SOLAR CONSTANT, ALBEDO AND THE REFLECTIVITIES FROM THE SATELLITES ARE VERY WELL MEASURED.

•THERMAL DRAG, YARKOVSKI AND RUBINCAM EFFECTS, ARE MODELLED IN GEODYN, AND CONTRIBUTE WITH A SMALL ERROR OF A FRACTION OF 1%.

•PARTICLE DRAG MISMODELLING IS NEGLEGIBLE IN THE LAGEOS MEASUREMENTS AND SUBSTANTIALLY NEGLEGIBLE IN THE LARES EXPERIMENT BECAUSE (1) **LARES HAS THE HIGHEST MASS-TO-CROSS-SECTIONAL RATIO OF ANY OTHER KNOWN OBJECT IN THE SOLAR SYSTEM** (IT IS MADE OF A TUNGSTEN ALLOY) AND (2) BECAUSE LARES CONTRIBUTES TO THE MEASUREMENT OF THE LENSE-THIRRING EFFECT FOR A 10 % ONLY OF THE MEASURED VALUE OF THE LENSE-THIRRING EFFECT, (3) BECAUSE THERE IS A **WELL KNOW RESULT OF CELESTIAL MECHANICS THAT SHOWS THAT THE NODE OF A SATELLITE IS NOT DISPLACED AT ALL BY PARTICLE DRAG, EVEN BY ASSUMING THAT THE ATMOSPHERE IS ROTATING AT THE LARES ALTITUDE AND (4) BECAUSE THE EFFECT ON INCLINATION AND SEMIMAJOR AXIS CAN VERY ACCURATELY BE MEASURED BY SLR (Contrary to some false statements that you can find in the literature: Iorio arXiv:0809.3564v1 [gr-qc] 21 Sep 2008).**



LAGEOS 1	i [μas]	Ω [μas]
Minimum	0	0
Maximum	30	50
Points	156	156
Median	10	10
RMS	16	13
Std Deviation	8.6	7.8

_KEP_rms_2000-2002

Differences in the inclinations of LAGEOS and LAGEOS 2 between Marini-Murray (MM) and the new Mendes-Pavlis (MP) model

The differences between the two atmospheric refraction models are at the level of 10 microarcseconds for the inclination, corresponding to about 0.2 % of the Lense-Thirring effect

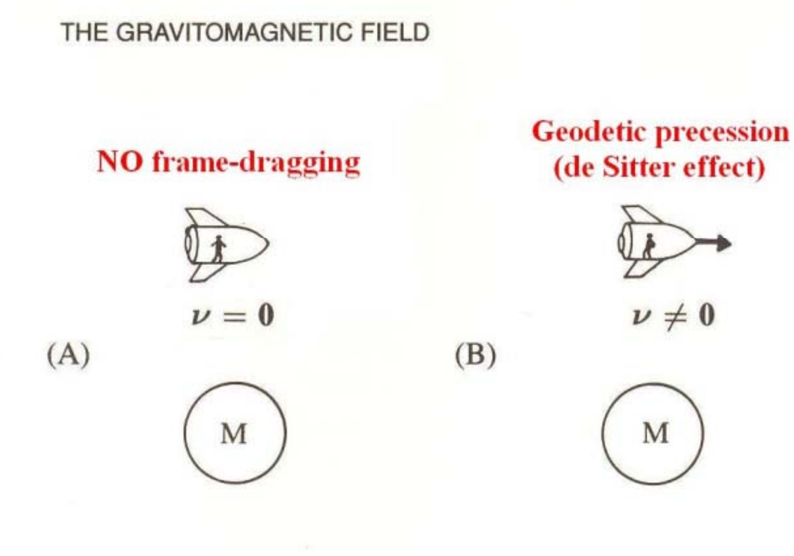
In addition, the modeling errors in the inclination are routinely corrected for using the inclination residuals using the formula: $\delta\Omega = (\partial\Omega / \partial I) * \delta I$, where δI are the inclination residuals of LAGEOS, LAGEOS 2 and LARES determined with GEODYN and EPOS-OC over each arc, i.e., for LAGEOS: $\delta\Omega = 6 * \delta I$ * yr^{-1}, and for LAGEOS 2: $\delta\Omega_2 = 5.3 * \delta I$ * yr^{-1}, where δI is measured in milliarcsec.

Has the gravitomagnetic field been measured by Lunar Laser Ranging?

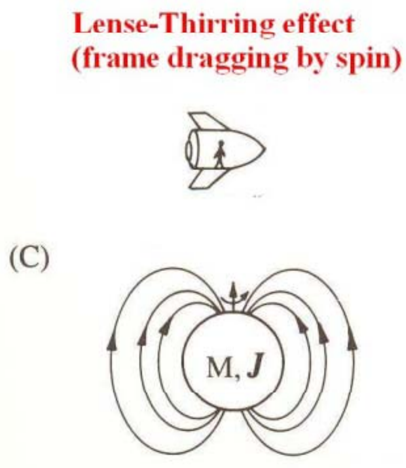
Murphy, Nordtvedt, Turyshev ←→ Kopeikin, I.C.

- Soffel, Klioner, Mueller, Biskupek (fitted the LLR data for a gravitomagnetic parameter η , measured with some ~ 10^{-3} accuracy)

- Kopeikin useful distinction between translational and intrinsic gravitomagnetic effects



The geodetic precession **IS** indeed a gravitomagnetic effect (frame-dragging or **B** ~ **v** x **E**) measured with some ~ 10^{-3} accuracy by LLR

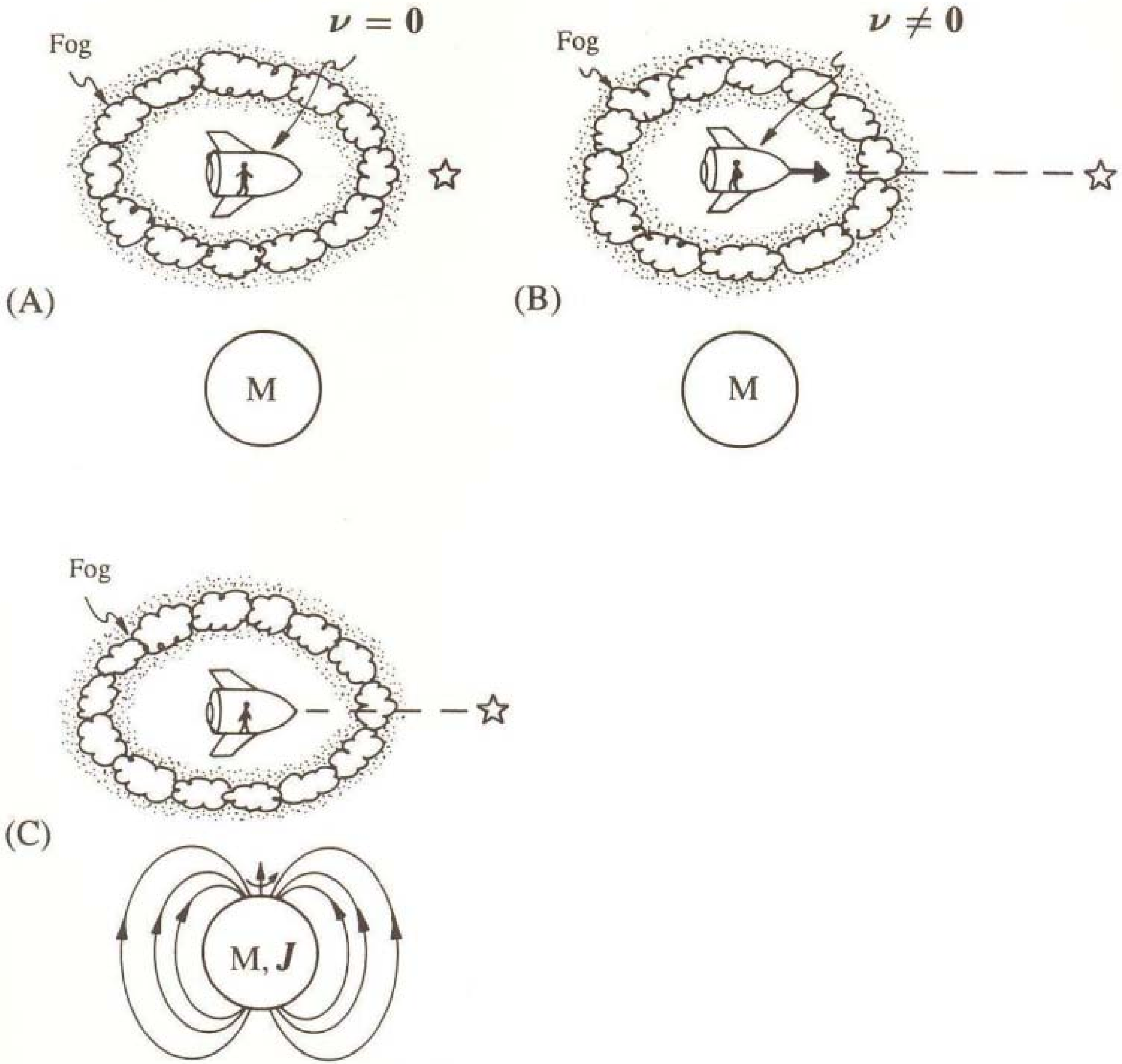


Only in this case (c) additional spacetime curvature is generated by the spin of the central body (Kerr geometry).

But how can we define it?

Not by looking at the g_{0k} non-diagonal components of the metric, nor by simply looking at the magnetic-like components of the Riemann tensor R_{i0kl}

THE GRAVITOMAGNETIC FIELD



INVARIANT CHARACTERIZATION of "INTRINSIC" GRAVITOMAGNETISM

Gravitomagnetism defined without approximations by the Riemann tensor in a local Fermi frame.
Matte-1953

By explicit spacetime invariants built with the Riemann tensor:
I.C. 1994
I.C. and Wheeler 1995:
for the Kerr metric:

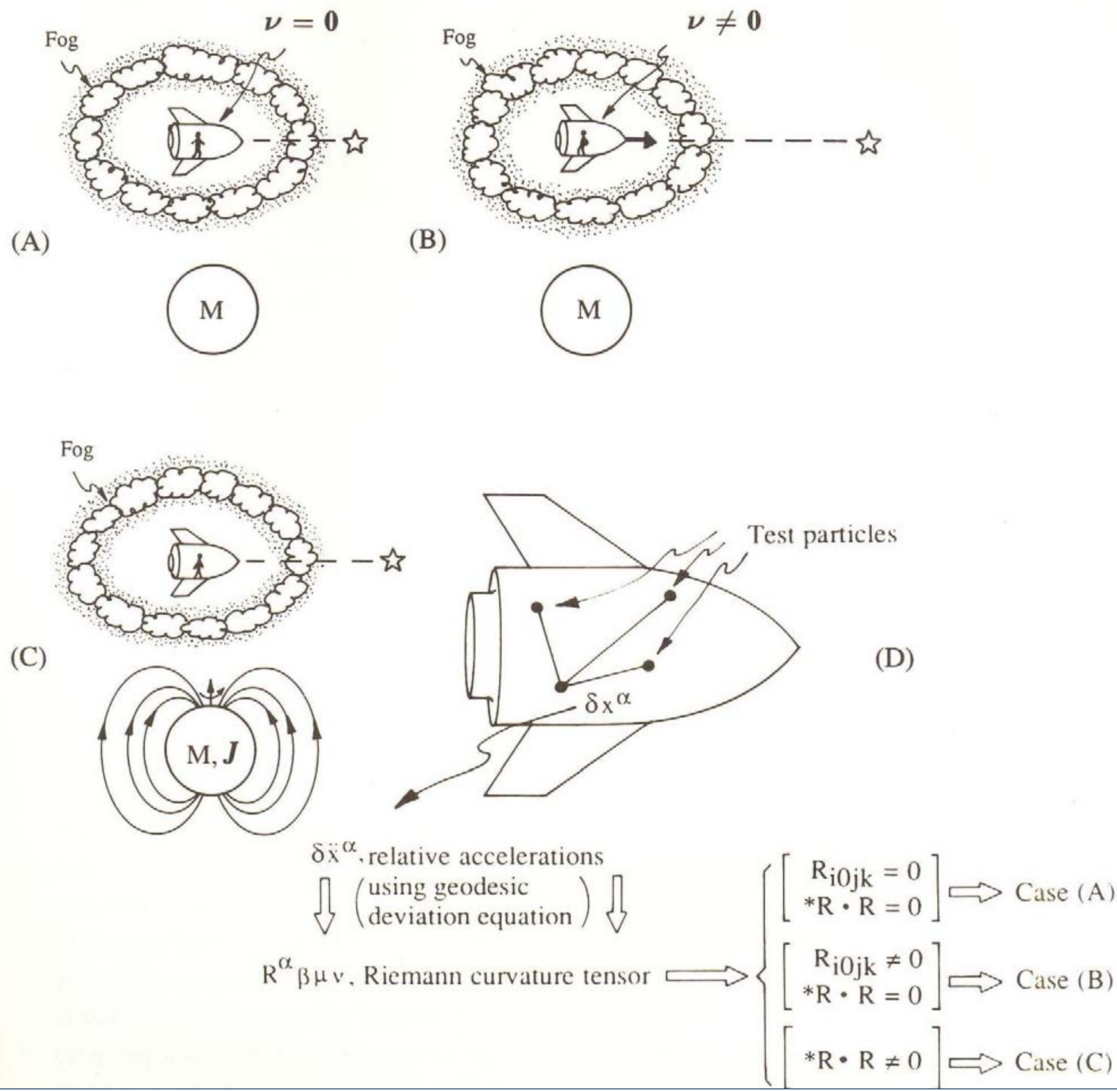
$$\tfrac{1}{2}\,\epsilon_{\alpha\beta\sigma\rho}\,R^{\sigma\rho}{}_{\mu\nu}\,R^{\alpha\beta\mu\nu} = 1536\,J\,M\cos\theta\,(\rho^5\rho^{-6} - \rho^3\rho^{-5} + 3/16\,\rho\,\rho^{-4})$$

In weak-field and slow-motion:

$$^{*}R \cdot R = 288\,(J\,M)/r^7 \cos\theta \;+ \cdots$$

J = a**M** = **angular momentum**

*R · R similar to ***F · F** in electrodynamics
Similarly ***R · R** is different from zero in the case of two **massive** bodies moving with respect to each other (calculated using the PPN metric).



$\nu = 0$
Fog
(A)
M
$\nu \neq 0$
Fog
(B)
M
Fog
(C)
M, J
Test particles
δx^α
(D)
$\delta\ddot{x}^\alpha$, relative accelerations
(using geodesic deviation equation)
$R^\alpha{}_{\beta\mu\nu}$, Riemann curvature tensor
$R_{i0jk} = 0$
$^*R \cdot R = 0$
Case (A)
$R_{i0jk} \neq 0$
$^*R \cdot R = 0$
Case (B)
$^*R \cdot R \neq 0$
Case (C)

“Intrinsic” Gravitomagnetism and Lunar Laser Ranging

In order to distinguish between 'translational' gravitomagnetic effect and “intrinsic” gravitomagnetic effects, let us calculate the invariant *R · R

***R · R** is formally similar to the electromagnetism invariant ***F · F** ~ **E · B**

(**E** and **B** are electric and magnetic fields)

In weak-field and slow-motion, we have:

$$^{*}\mathbf{R}\cdot\mathbf{R} = 288\,\frac{\mathrm{M}_{\oplus}\mathrm{M}_{S}}{r_{M}^{4}\,r_{M\oplus}^{4}}\,z_{M}\left(v_{\oplus}^{x}\,y_{\oplus} - v_{\oplus}^{y}\,x_{\oplus}\right)\left(\hat{x}_{M}\cdot\hat{r}_{M\oplus}\right) + \ldots$$

and thus on the ecliptic plane: ***R · R** = **0**.

Similarly to electrodynamics, indeed ***R · R** ~ **G · H** but **H** ~ **v x G** and thus ***R · R** ~ **G · (v x G)** = **0 ; Similar to *F · F** ~ **E · B** ~ **E · (v x E)**

This gravitomagnetic invariant is null on the ecliptic plane and substantially null on the Moon orbit: I.C. arXiv:0809.3219v1 [gr-qc] 18 Sep 2008 (to be published)

Uncertainty in J4-dot (secular variation in J4)

THE UNCERTAINTY DUE TO J4-DOT MISMODELLING IS LESS THAN 1%.

- I.C. and E.C. Pavlis, New Astronomy, **10** 636 (2005).
- I.C., E.C. Pavlis and R. Peron, New Astronomy,
11 527 (2006).
- M.K Cheng and B.D. Tapley, J. Geophys. Res.
109 9402 (2004).
- D. Lucchesi, Int. J. Mod. Phys. D, 2005.
- See Poster
- J. Ries "Private Communication" (2006):

"The results show that with the latest generation of GRACE models appear to support a detection of the Lense-Thirring effect at about the 15 percent level." and "If we allow some reduction due to averaging across the various solutions, the error is reduced to approximately 7%." and:
"We also note that removing the rates for J3, J4 and J6 from the analysis has a negligible effect."

GRAVITATIONAL ERRORS

Largest gravitational perturbations of the orbits of the LARES and LAGEOS satellites are due to the non-sphericity of the Earth gravity field (described by the expansion of the Earth potential in spherical harmonics).

The only secular effect affecting the orbit of a satellite are due to the **axially symmetric deviations from spherical symmetry of the Earth gravitational potential described by the even zonal harmonics, i.e., the spherical harmonics of the Earth gravitational potential of even degree and zero order**, e.g., the C20 harmonic describing the well known quadrupole moment of Earth.

The periodical effect of the K1 tide can be averaged and fitted for (the period of the perturbation is well known and is the period of the LARES node) over many cycles, the LARES nodal line has a complete revolution every 7.65 months. The final error, over a period of a few years of analysis, is less than 1 %.

GRAVITATIONAL ERRORS

There have been some false statements and wrong calculations in the literature (Iorio: arXiv:0809.1373v1 [gr-qc] 8 Sep 2008) claiming that the total error due to higher harmonics is very large, i.e., of about 100 or 1000 % the Lense-Thirring effect!!!

These calculations are completely wrong by three orders of magnitude:

(1) See the ISSI-ESA workshop (Bern October 08) poster available online

(2) The detailed calculations that will be available at the LARES web site

(3) The detailed calculations are being published in the book:
"John Wheeler and General Relativity" and in a forthcoming journal paper.

(4) From a "poor man" approach, it is clear that this type of error decreases with the degree of the harmonic, indeed:

$$\delta\Omega = K_2\, C_{20} + K_4\, C_{40} + \ldots + K_L\, C_{L0} + \ldots$$

however:

$K_L \sim R^L/a^{L+3/2} \ldots$

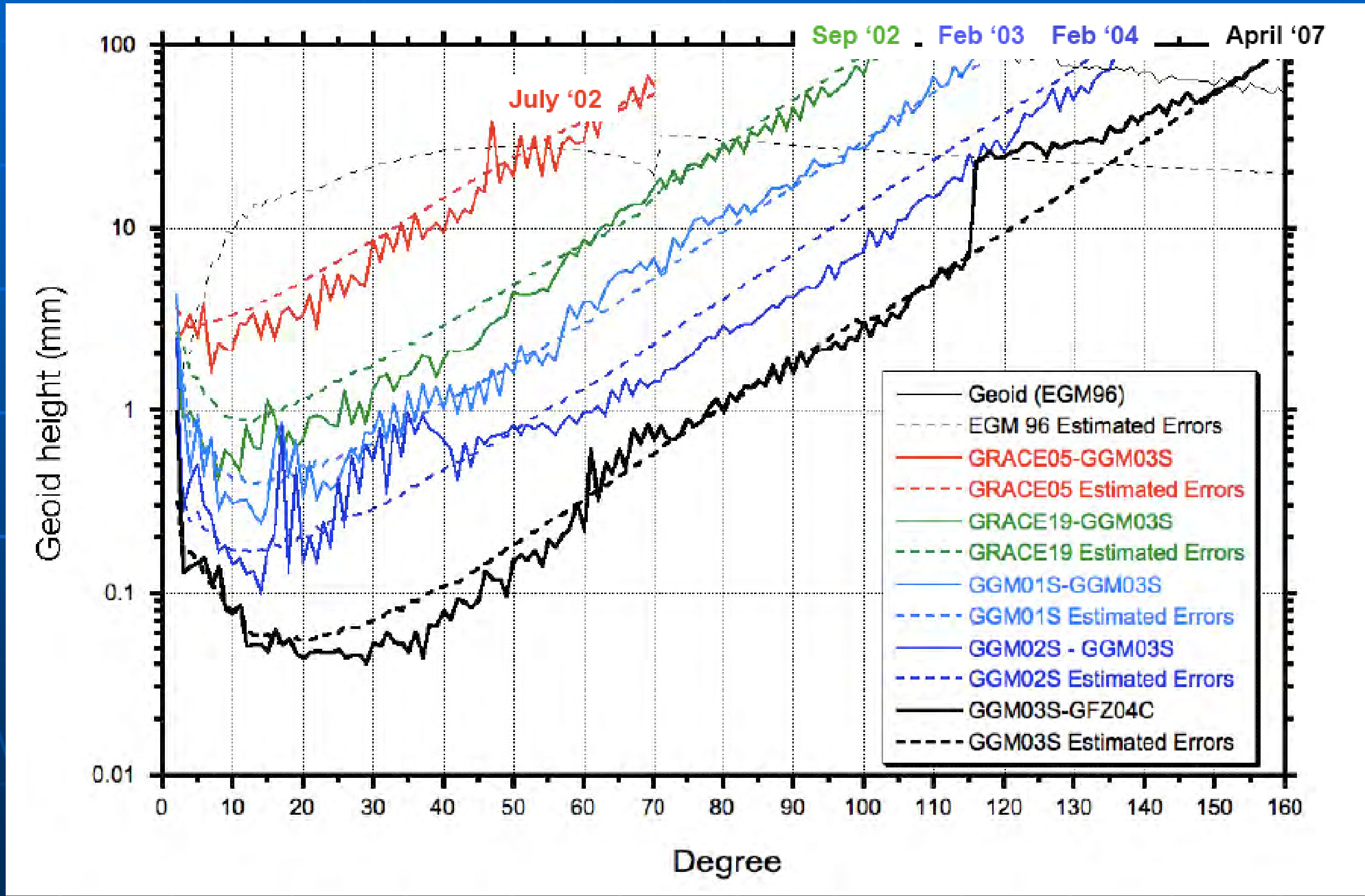
for example:

$K_2 / K_{20} \sim R^L/a^{L+3/2} = 2.5 * 10^{-2}$

$K_2 / K_{70} \sim R^L/a^{L+3/2} = 8.9 * 10^{-7}$

GRAVITATIONAL ERRORS

By comparing and differencing different GRACE Earth gravity field models we can conclude that their published uncertainties are within two or at most three times the differences of theif coefficients.



Successive static gravity model improvement from GRACE data as a function of the data span used in the solution and correlation of the model errors and the assumed calibrated model errors [Tapley et al., 2007].

Time dependent Earth gravity field

- Tides: uncertainty of the order of 1 %
- Secular time variation of J_2-dot, J_4-dot, ...

Used: J_2-dot= -2.6×10^{-11}/yr   Cox & Chao Science2002

J_4-dot= -1.41×10^{-11}/yr +/- 0.6×10^{-11}/yr

δ(J_2-dot) cancelled using our combination of nodes

and the 2005 value of J_4-dot is:

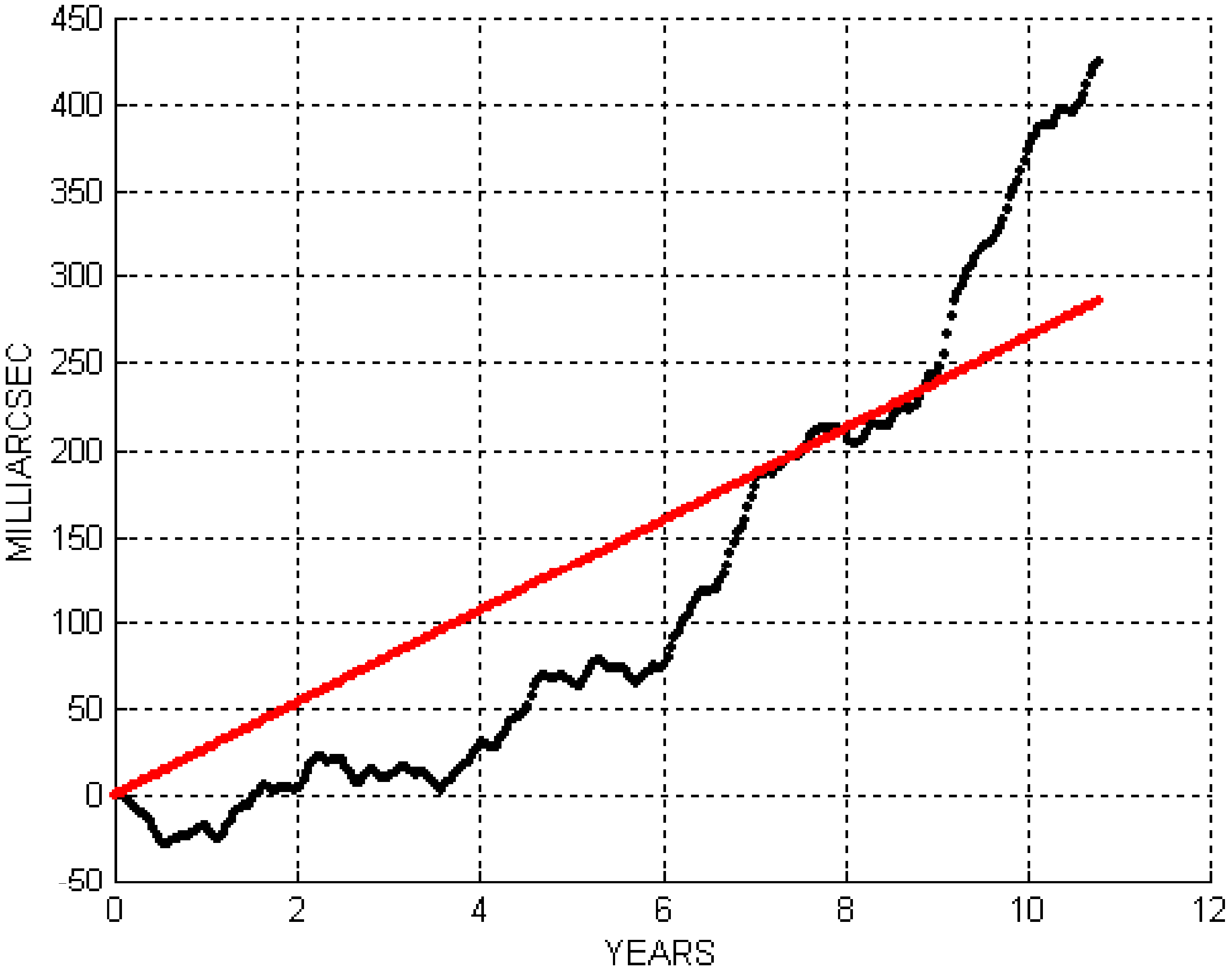
J_4-dot= -1.99×10^{-11}/yr   Cheng & Tapley 2004:

These 2005 values of J_{2n}-dot using GEODYN imply an error of about 6.9 % of the Lense-Thirring effect (without fitting the residuals with a parabola) and a total error due to time-dependent gravity uncertainties of abour 8 %.

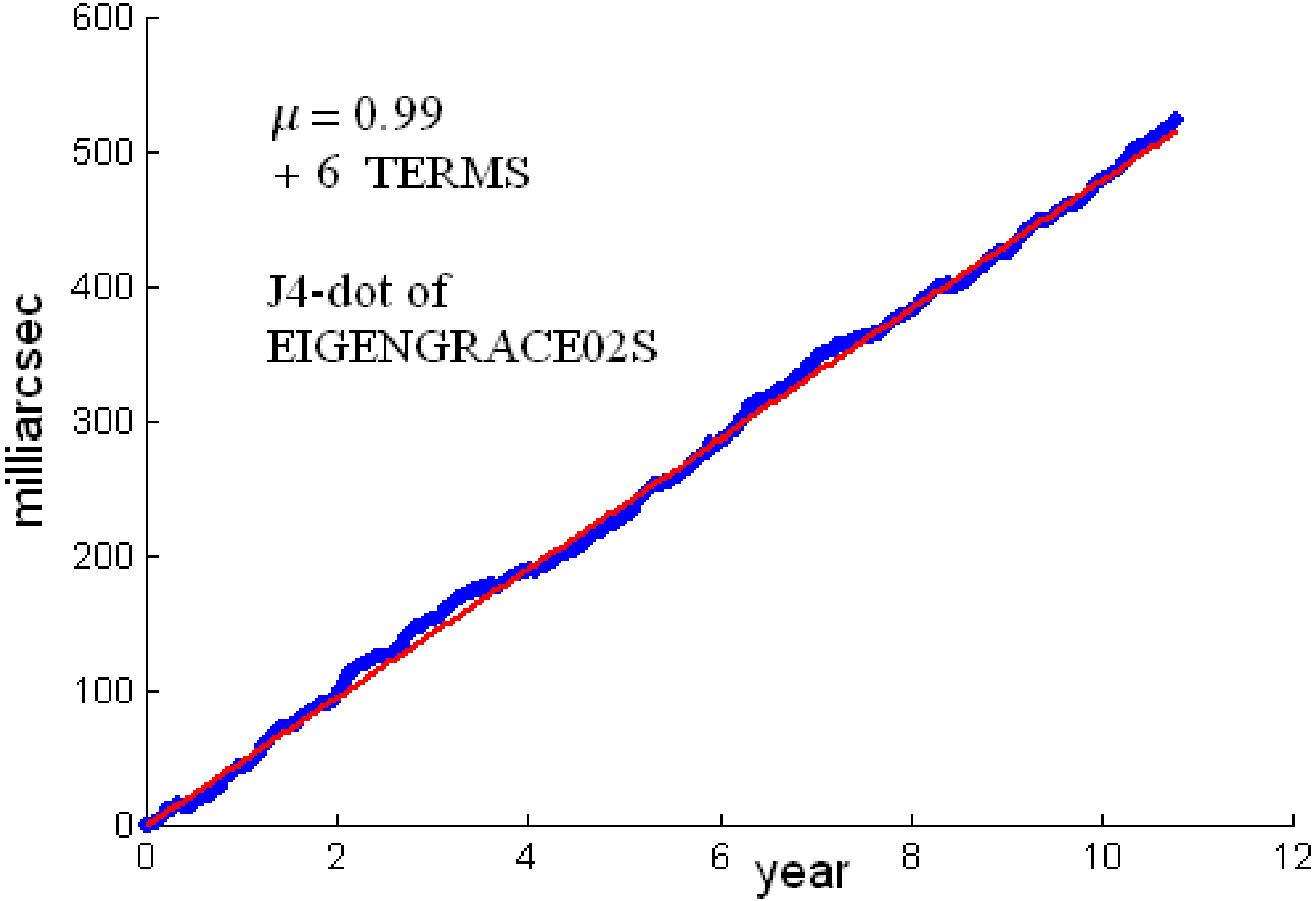
However, since the effect of the J_{2n}-dot on the nodal longitude of the LAGEOS satellites produces an effect quadratic in t, to bound the error due to the J_{2n}-dot, we can simply fit our observed rediduals for a linear trend t plus a quadratic term $t^{2.}$ The result is:

Using or NOT a J_4-dot effect in our data-analysis of the LAGEOS satellites and fitting a straight line plus a parabola gives at most a 1% change in the measurement of the GR prediction.

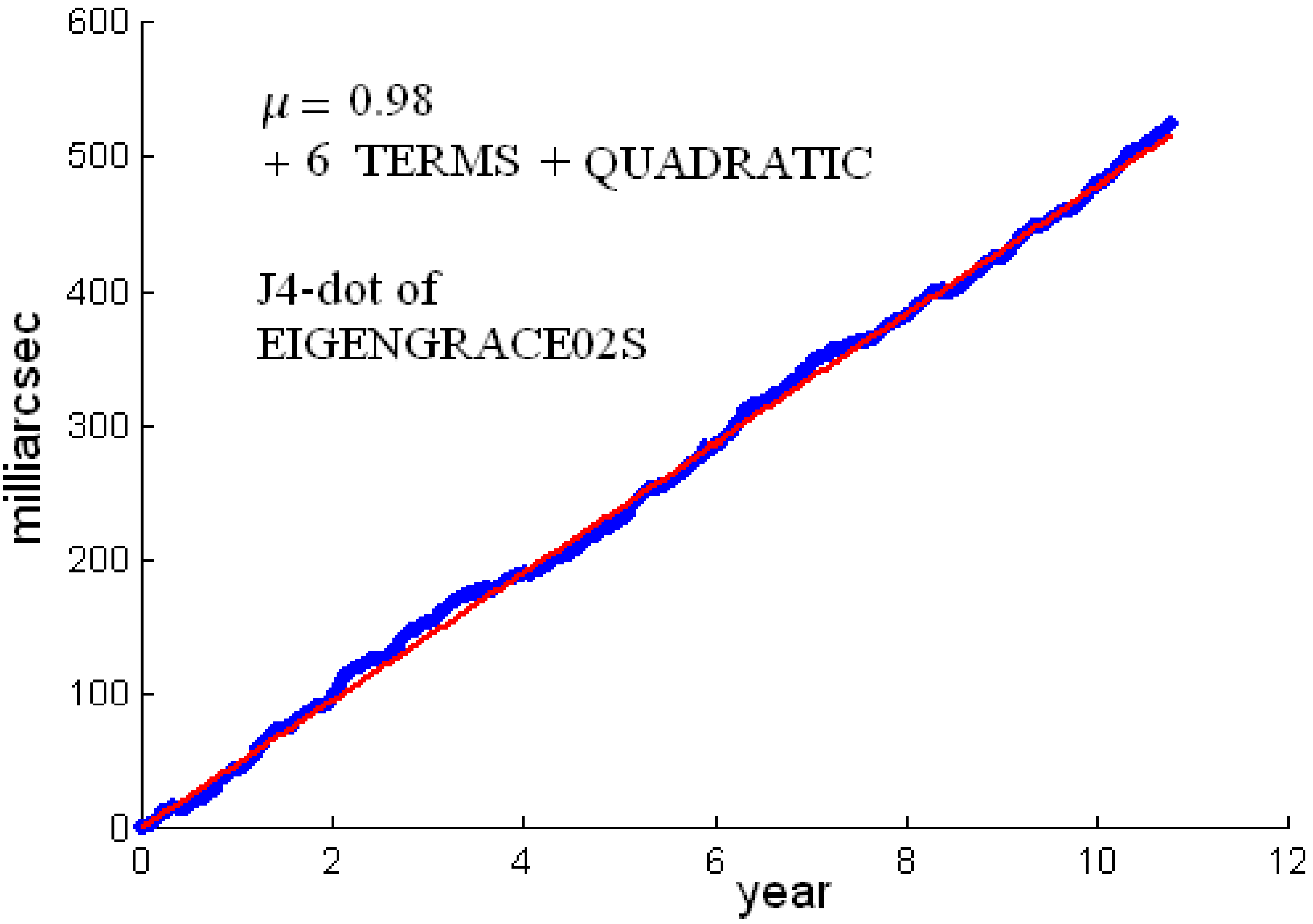
Therefore the total error due to the J_{2n}-dot is 1 % of the Lense-Thirring effect in our determination of the Lense-Thirring effect



EFFECT OF MISMODELLING OF J4-DOT
ON THE LENSE-THIRRING MEASUREMENT

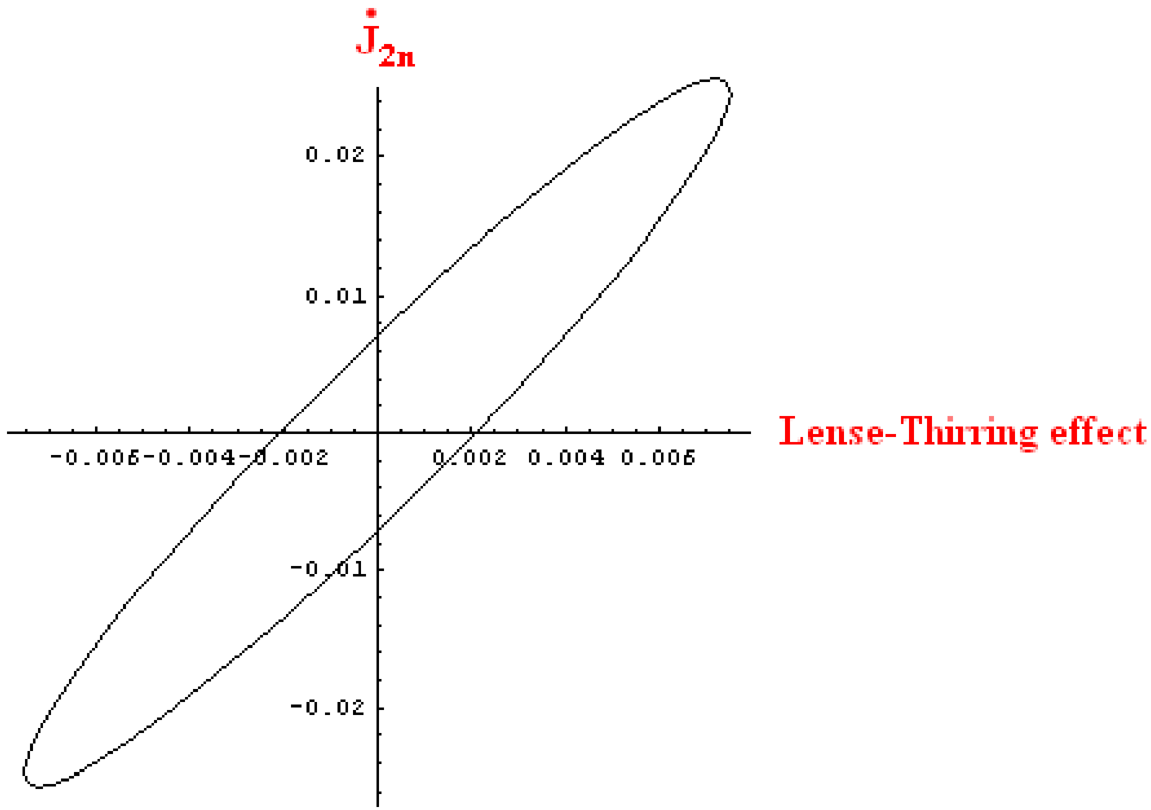


$\mu = 0.99$
$+ 6$ TERMS
J4-dot of
EIGENGRACE02S
milliarcsec
600
500
400
300
200
100
0
0
2
4
6
8
10
12
year



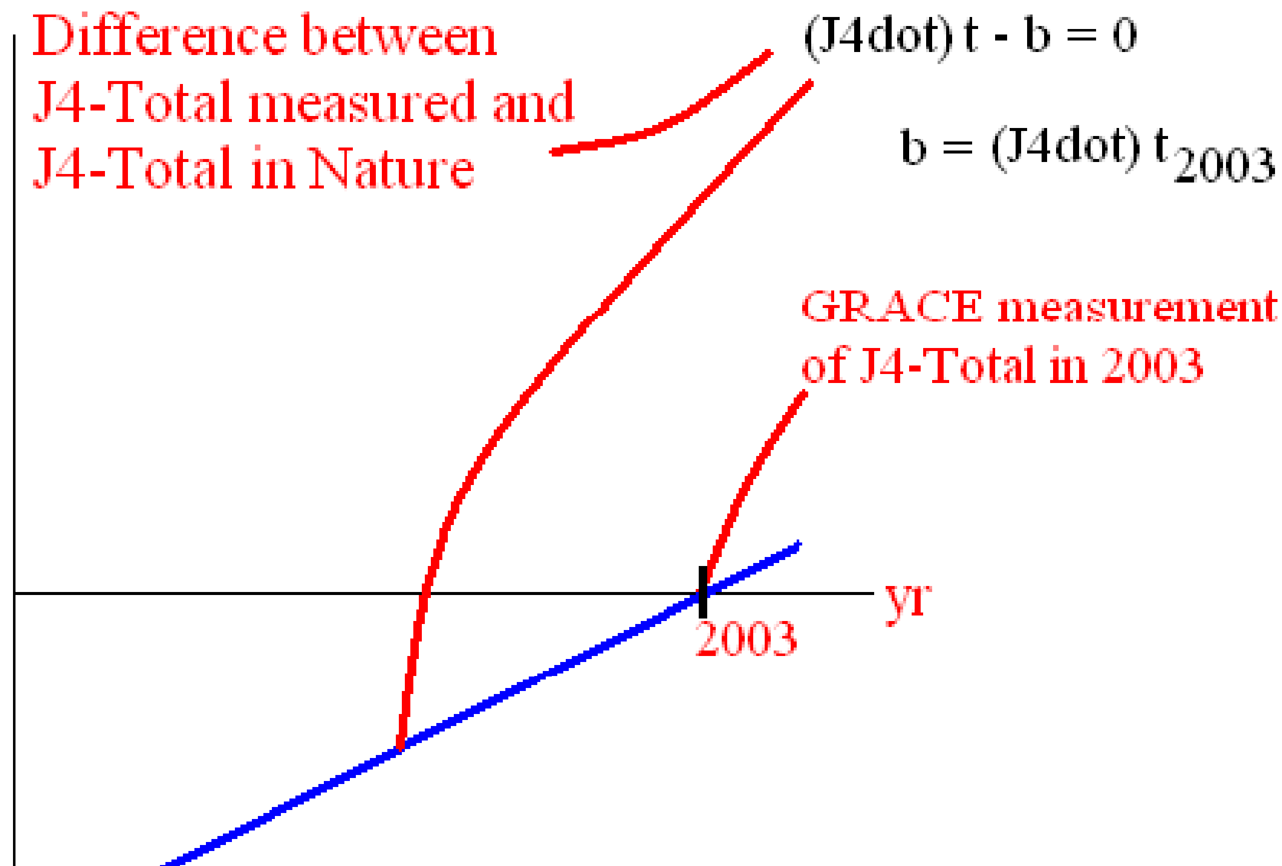
$\mu = 0.98$
+ 6 TERMS + QUADRATIC
J4-dot of
EIGENGRACE02S
milliarcsec
year
0
100
200
300
400
500
600
0
2
4
6
8
10
12

Confidence ellipse for a 99 % probability of fit



Units: x-axis: 1 = Lense-Thirring effect in the combination of the LAGEOS nodes, 48.2 mas/yr

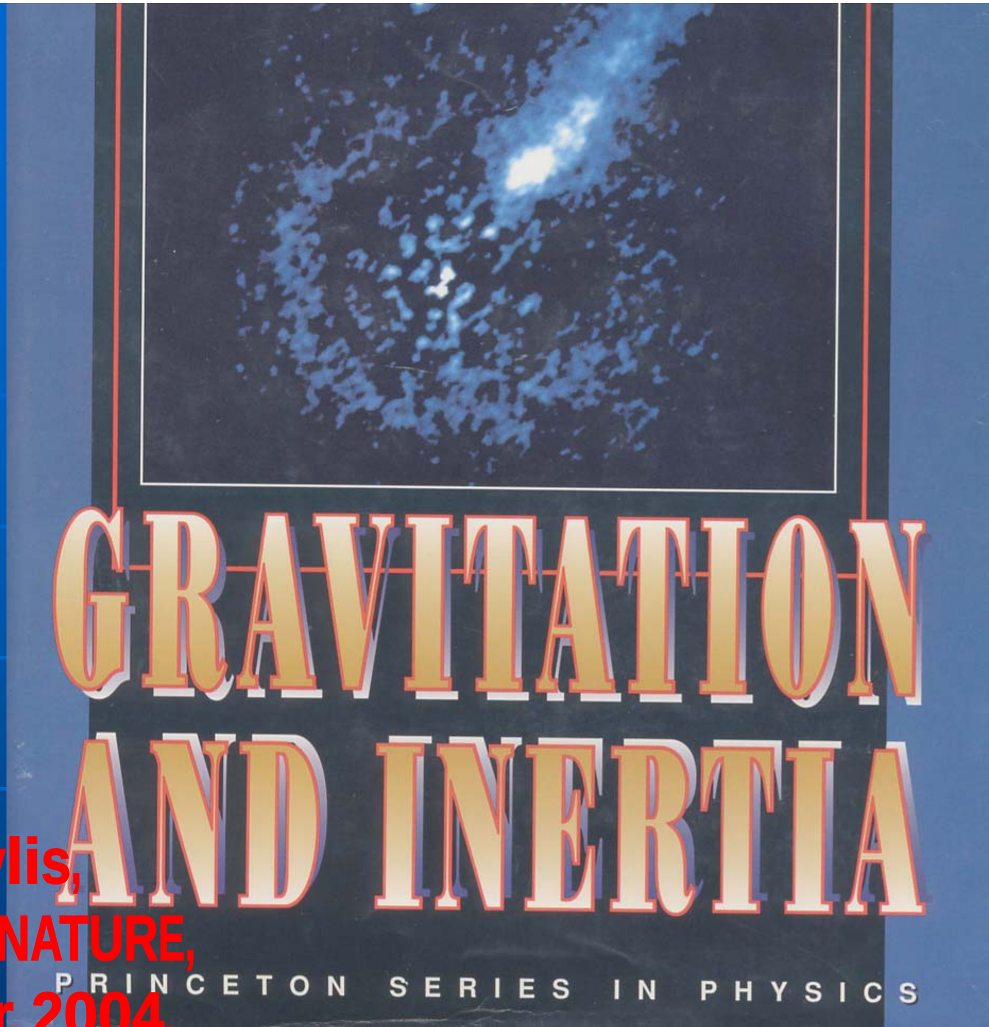
y-axis: 1 = 1.41 10^{-11}/yr



Integrando per ottenere l'effetto residuo sulla longitudine del nodo (cumulativa) si ottiene:

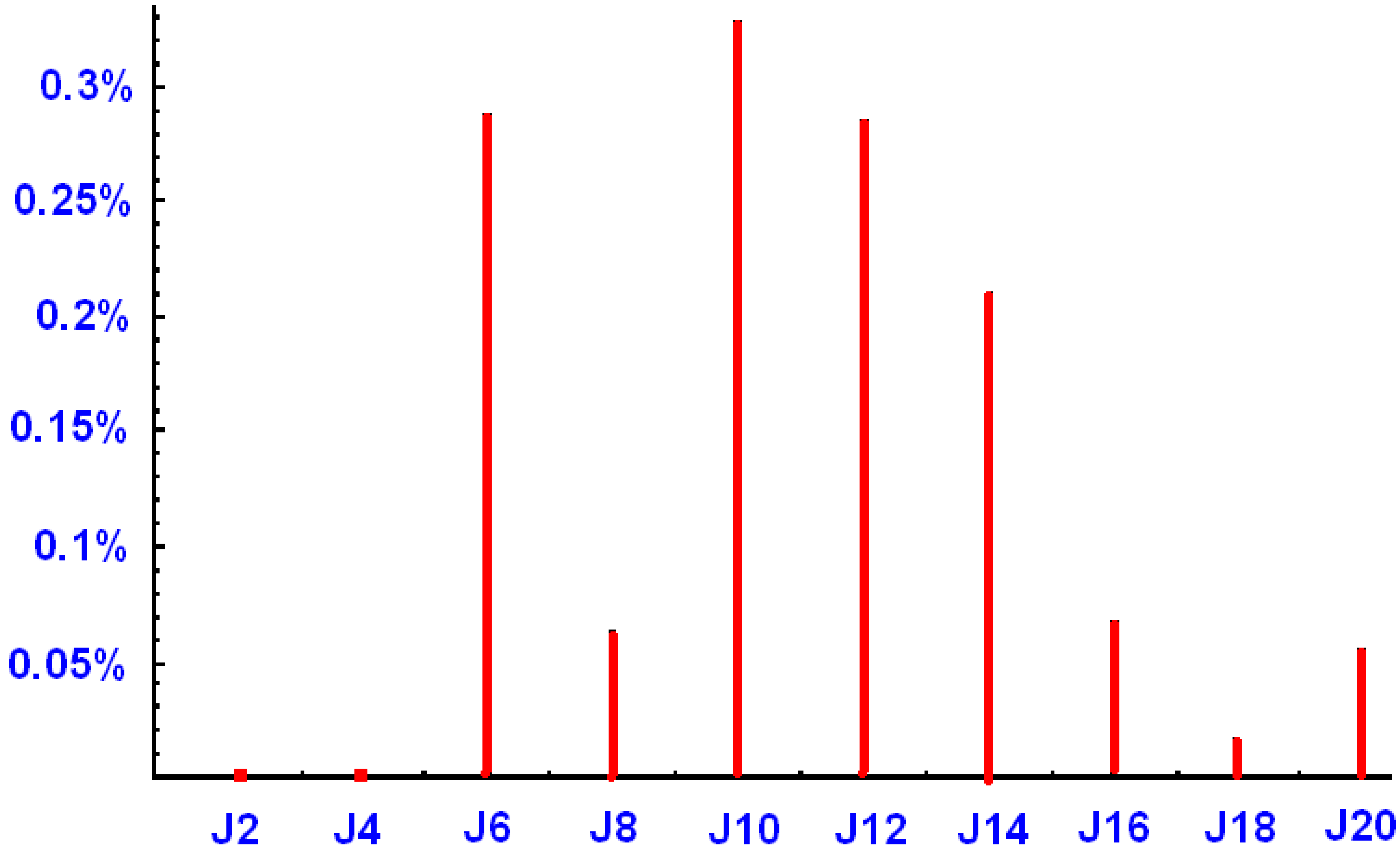
$$(J4dot)\frac{\Delta t^2}{2} + b\,\Delta t + LT\,\Delta t$$

Ma b = (J4dot) t_{2003} quindi si puo' semplicemente fittare una parabola più una retta !



I.C. & E.Pavlis,
Letters to NATURE,
21 October, 2004.

I.C., E.Pavlis and R.Peron,,
New Astronomy
2006.



Percent error (relative to the Lense-Thirring effect) due to each even zonal harmonic using LARES, LAGEOS and LAGEOS 2

The measurement reported in Nature uses the NODES of the two laser ranged satellites LAGEOS and LAGEOS 2 to cancel the effect of the first even zonal harmonic coefficient J_2 of Earth and to measure the Lense-Thirring effect. Furthermore, it uses the accurate model EIGENGRACE02S of the Earth gravity field developed by GFZ of Potsdam with the data of the NASA GRACE satellites.

The **idea** of using the **nodes of two laser ranged satellites of LAGEOS type to measure the Lense-Thirring effect** was published for the first time by **I.C. in 1984-86** (see e.g.: "Measurement of the Lense-Thirring drag on high-altitude laser-ranged artificial satellites", **I. C., *Physical Review Letters*, 56, 278-281, 1986**).

The **idea** to **use the nodes of *n* satellites of LAGEOS type to cancel the effect of the first *n*-1 Earth even zonal harmonics and to measure the Lense-Thirring effect** was published for the first time in 1989 by I.C. (see e.g.: "A comprehensive introduction to the LAGEOS gravitomagnetic experiment: from the importance of the gravitomagnetic field in physics to preliminary error analysis and error budget", **I. Ciufolini, Int. Journ. of Phys. A, 4, 3083-3145, 1989**. The measurement of the Nature paper is simply the case of *n*=2. "**A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc, and one satellite to measure $(\Omega\text{-dot})^{\text{Lense-Thirring}}$**".

Thus, the case of the Nature paper is just the one with two satellites. At that time the error due to the even zonal harmonics was quite larger due to the much less accurate Earth gravity models and the LAGEOS 2 satellite was not yet launched (it was launched in 1992).

The **idea,** in particular, to use **the two nodes of the satellites LAGEOS and LAGEOS 2**, together with the perigee of LAGEOS 2, was published, together with the corresponding formula, for the first time in 1996 by I.C. (see, e.g.: "On A New Method to Measure the Gravitomagnetic Field Using Two Orbiting Satellites", **I. C., Il Nuovo Cimento A, 109, 1709, 1996**). See, e.g., the formula (15) on page 1717:

“$\delta(\Omega\text{-dot})_{I}+\delta(\Omega\text{-dot})_{II}+\delta(\omega\text{-dot})_{II} = \mu\,(31+31.5\,k_1-57\,k_2)$ milliarcsec/year + [other error sources (δC_{60}, δC_{80}, …, δI_{I}, δI_{II})],”

This is exactly eq. (1) of the Nature paper with k_2=0, i.e. there is no use of the perigee, but just of the nodes, according to what we suggested in the above 1989 I.C. paper and similar following papers.

The *results plus analysis plus the* ***combination*** for the measurement of the Lense-Thirring effect using the two nodes only of the LAGEOS satellites and some of the older models based on CHAMP and GRACE have also been presented and published by I.C. in the proceedings (IOP) of the Villa Mondragone School, Rome, 2002, presented by I.C. at a plenary talk at the Marcel Grossmann meeeting in Rio de Janeiro in July 2003 (to appear in the proceedings).

Lense-Thirring imprint on the GRACE models

It is a null effect, as shown in I.C. & E.P. 2005.

Indeed the Lense-Thirring effect is, at the post-Newtonian order, formally similar to the Lorents force, i.e., $\mathbf{a} = \mathbf{v} \times \mathbf{H}$, therefore for a polar satellite (H is contained in the polar orbital plane) **a** is orthogonal to its orbital plane.

On the contrary the acceleration resulting from the even zonal harmonics on a polar satellite (such as GRACE) is along-track (along v) and radial (r) but it has zero out-of-plane component.

Therefore, the two “forces” are orthogonal and in no way one can affect and bias the other: $a_{\mathbf{LT}} \cdot \mathbf{a}_{\mathbf{evenzonals}} = 0$.

Therefore, the influence of the Lense-Thirring effect in the determination of the even zonal harmonics with a polar satellite with circular orbit is zero, i.e., the influence of LT effect on the GRACE models is negligible.

Inclination Error

- Average Inclination Residual:
  for LAGEOS is -0.015 milliarcsec,
  for LAGEOS less than -0.002 milliarcsec
  On the node this inclination error is:
- for LAGEOS is
  less than .3 % Lense-Thirring effect
- for LAGEOS 2 is
  less than 0.04 % Lense-Thirring effect

Uncertainty due to Inclination Errors

We routinely correct our node residuals with the inclination residuals: δI_{I} and δI_{II} using the formulas (see I.C., Nuovo Cimento, 1996):

$$\delta\,\Omega_{\mathrm{I}} = \partial\,\Omega_{\mathrm{I}} / \partial\,I_{\mathrm{I}} \bullet \delta\,I_{\mathrm{I}} = 6 \bullet \delta\,I_{\mathrm{I}}$$

$$\delta\,\Omega_{\mathrm{II}} = \partial\,\Omega_{\mathrm{II}} / \partial\,I_{\mathrm{II}} \bullet \delta\,I_{\mathrm{II}} = 5.3 \bullet \delta\,I_{\mathrm{II}}$$

So, if there is any systematic bias left in the inclinations of both LAGEOS and LAGEOS II it must be due atmospheric refraction mismodeling. However, on the basis of the new refraction models of Mendes, Prates E. Pavlis, D. Pavlis and Langley, 2002, on a long period of time of about 11 years the *constant total* systematic *out-of-plane* error is negligible (the nodal plane has many revolutions); even if we assume that over a very long period of observation we still have a *constant total* systematic *out-of-plane* error in the inclination of 3 mm (of the order of the uncertainty of the two-colour laser-ranging), that is a pessimistic hypothesis over our 11 years of observation, such an error would correspond to an inclination error of 0.05 milliarcsec and thus to an error of about 1 % only in the modelling of the nodal rate: already included in our final error budget in the 2% error due to random and stochastic errors .

Results.

Several points are clear. The LT estimates from the various models are all consistent with the GR prediction to within about 30% maximum or about 17% 1-sigma. The mean across all the models used here agrees with GR to 1%. If we allow some reduction due to averaging across the various solutions, the error is reduced to approximately 7%. Comparing the case where LT was modeled for GGM02S to the case where it was not modeled, the difference is exactly 1.00, indicating that the method is clearly sensitive to the modeling (or lack of modeling) the LT effect. A similar test was conducted regarding the effect of geodesic precession (de Sitter precession). This effect is roughly 50% of the LT effect, and failure to model it leads to a roughly 50% error in the LT estimate. We also note that removing the rates for J3, J4 and J6 from the analysis has a negligible effect, whereas failure to map J4 to a consistent epoch is much more significant (12%).

Finally, we note that the scatter in the estimates for C40 and C60 are significantly larger than the error assigned to these coefficients. In the case of C40, all coefficients were mapped to the same epoch, yet the scatter is larger than even the most pessimistic error estimate. When estimating the expected uncertainty in the LT experiment due to these harmonics, a more pessimistic error estimate should be used rather than those in the gravity model solutions.

Some conclusions by John Ries of the Center for Space Research of the University of Texas at Austin.

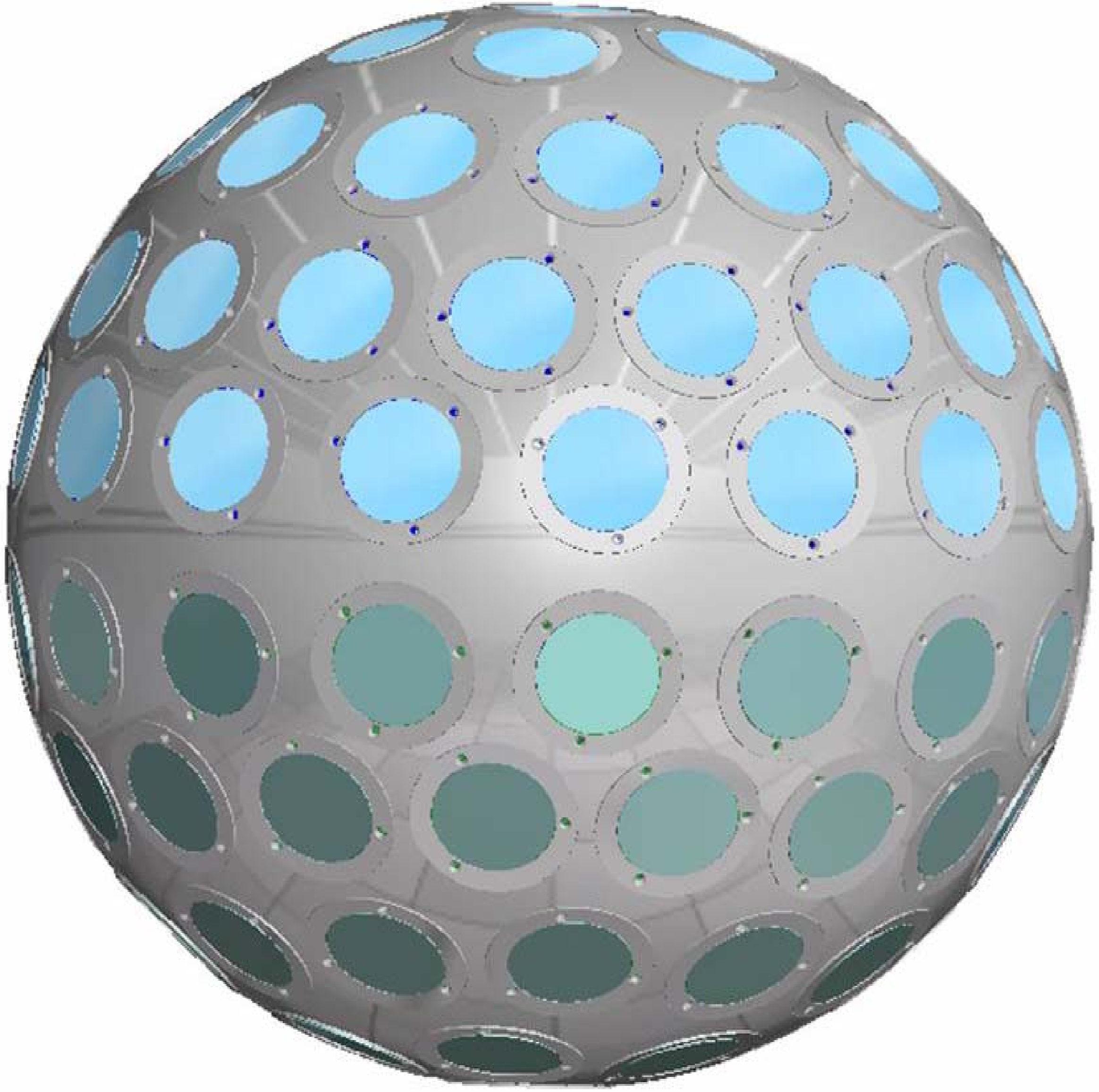
Introduction:

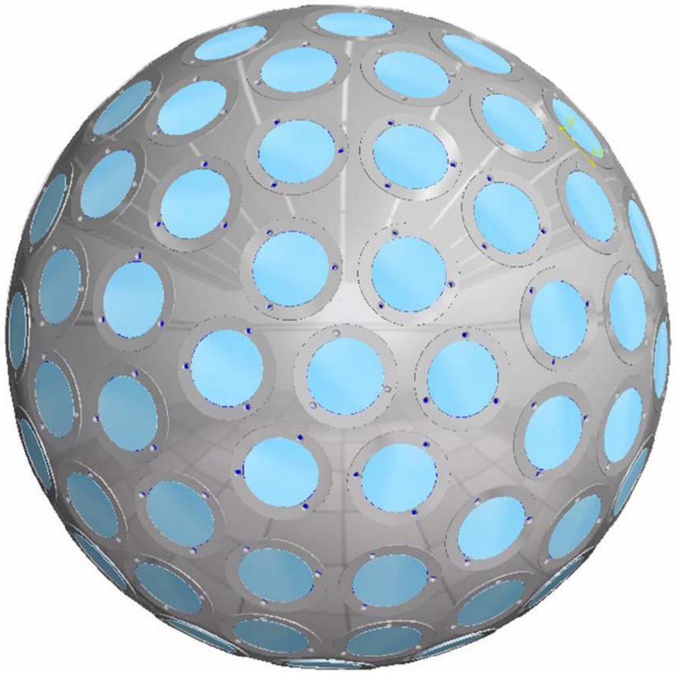
The principal goal was to attempt to validate the earlier published results using a wider variety of GRACE-based gravity models that are now available. This would provide a more confident error assessment. In addition, some sensitivity tests were conducted regarding the modeling of important related effects, and no important limitations were observed. The results show that with the latest generation of GRACE models appear to support a detection of the Lense-Thirring effect at about the 15 percent level. This relativistic test will continue to improve as the the GRACE-based gravity models incorporate more data and the processing methods improve.

Method:

The analysis followed the procedure outlined in Ciufolini et al. 1998 (for the node-node-perigee combination) and Ciufolini and Pavlis (2004) for the node-node combination. LAGEOS-1 and LAGEOS-2 satellite laser ranging (SLR) data covering the span of October 1992 through April 2006.

Several ‘second-generation’ GRACE-based gravity models were tested. These included GGM02S (Tapley et al., 2005), EIGEN-CG02S (Reigber et al., 2005), EIGEN-CG03C (Förste et al., 2005), EIGEN-GL04C (Förste et al., 2006), an unpublished gravity model (JEM04G) from the Jet Propulsion Laboratory based on 626 days of GRACE data (D. Yuan, personal communication, 2006).





LARES (Laser Relativity Satellite) to be launched by the Italian Space Agency in 2010 with VEGA (ASI-AVIO-ELV-ESA)

International Team

I. Ciufolini-Principal Investigator

- **University of Lecce and INFN Sezione di Lecce**
- **"Sapienza" University of Roma**
- **LNF-INFN**
- **University of Maryland**
- **NASA-Goddard**
- **University of Texas at Austin**
- **GFZ-Potsdam/Munich (GRACE team)**
- **AstroSpace Center of Lebedev Phys. Inst.-Moscow**

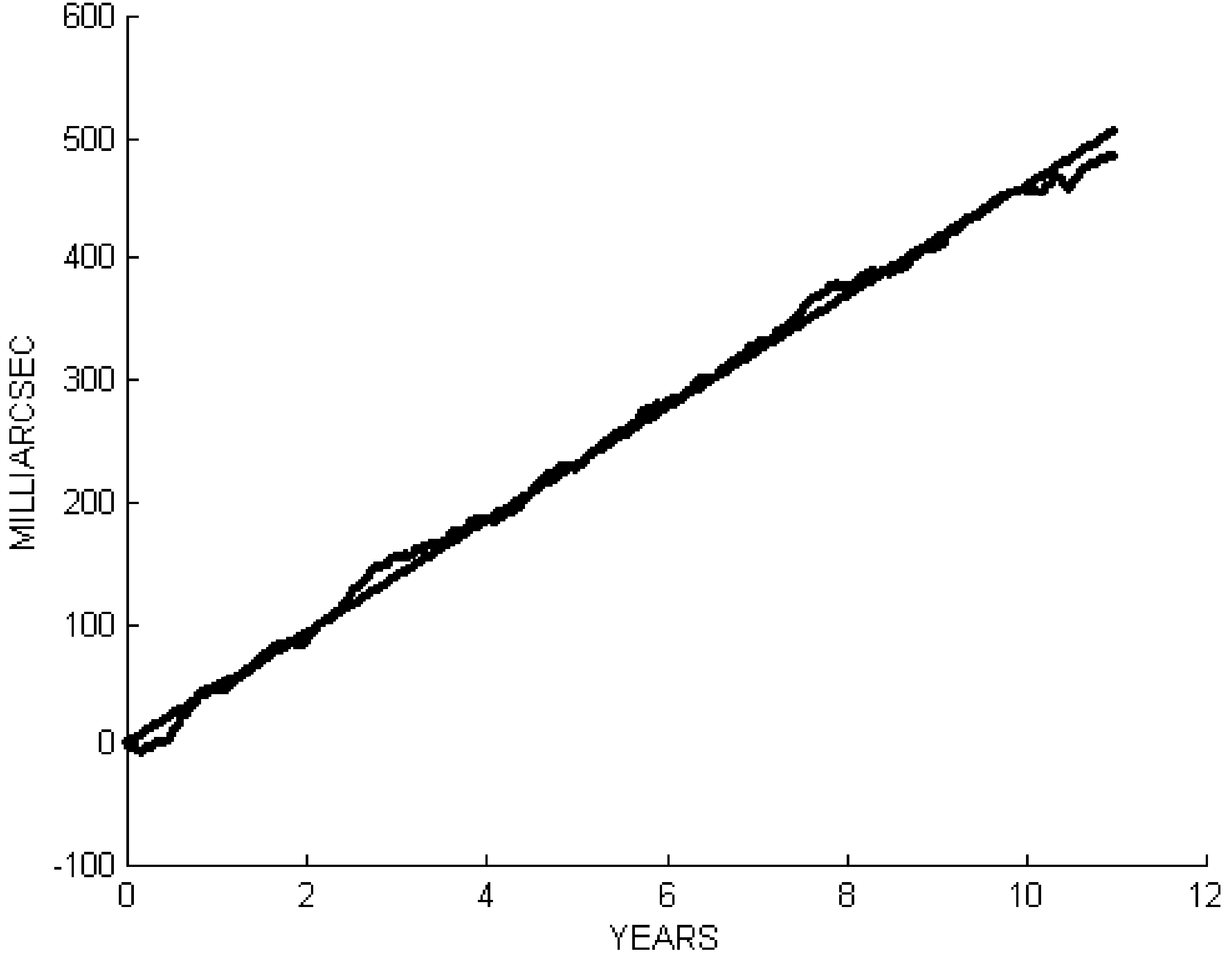
GRAVITATIONAL ERRORS

Confirming the Frame-Dragging Effect with Satellite Laser Ranging

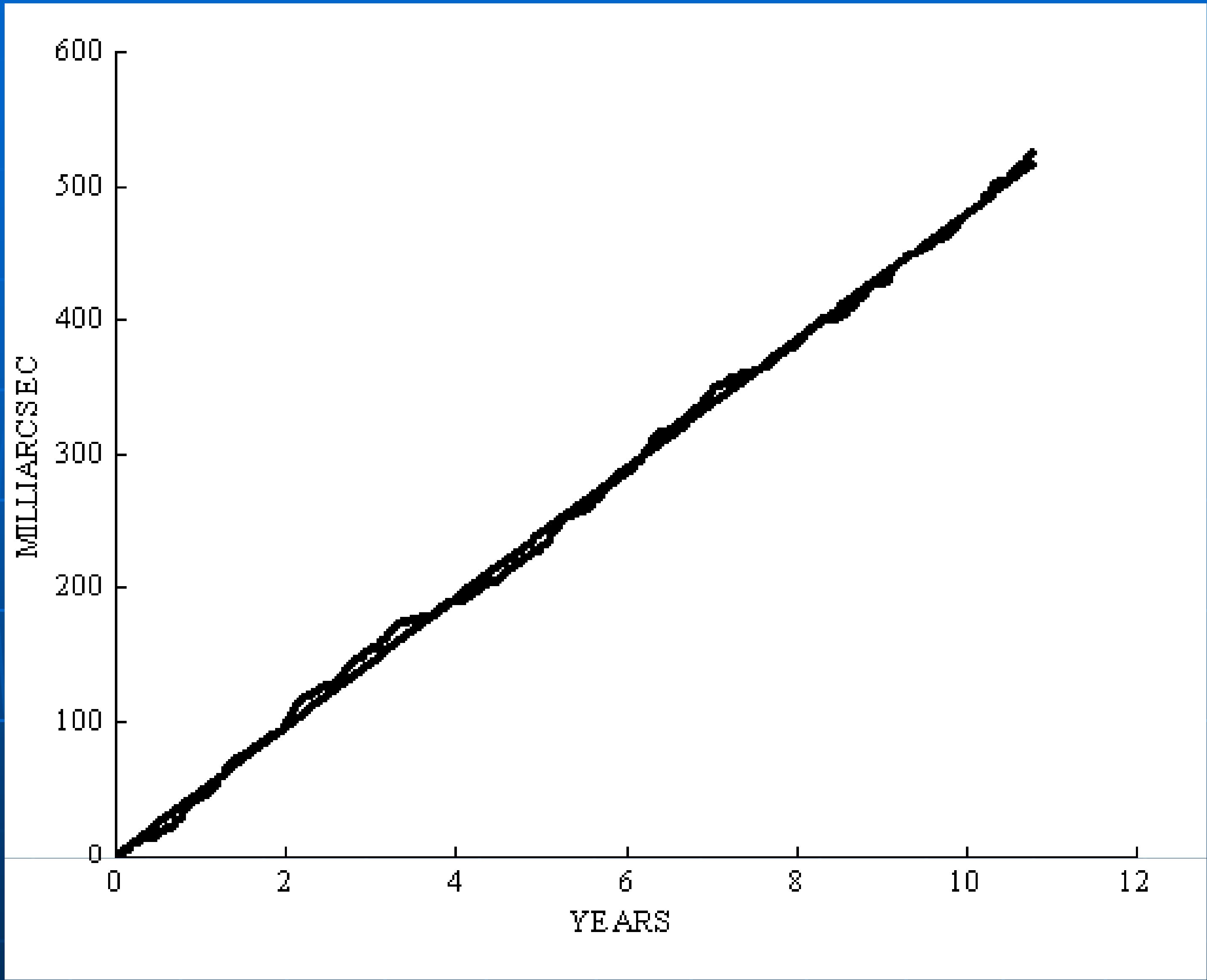
John C. Ries, Richard J. Eanes, Michael M. Watkins
The University of Texas at Austin, Center for Space Research, USA
E-mail: ries@csr.utexas.edu

The theory of General Relativity predicts several non-Newtonian effects that have been observed by experiment, but one that has proven to be challenging to directly confirm is the so-called 'frame dragging' effect. One manifestation of this effect is the Lense-Thirring precession of a satellite's orbital plane due to the Earth's rotation. While the signal is large enough to be easily observed with satellite laser ranging, the Lense-Thirring measurement uncertainty is limited by the knowledge of the even zonal harmonics of the Earth's gravity field that also produce Newtonian secular orbit precessions. In the late 1980's, it was proposed to launch the LAGEOS-3 satellite matching LAGEOS-1, except that the orbit inclination would be exactly supplementary to LAGEOS-1. This would have allowed the cancellation of the equal but opposite orbit precession due to the Earth's gravity field to reveal the Lense-Thirring precession. However, this satellite was never launched, and the orbit selected for LAGEOS-2 was not sufficiently close to the proposed LAGEOS-3 orbit specifications to support an accurate Lense-Thirring experiment with the available gravity models. However, this problem has been largely overcome with the dramatically improved models resulting from the joint NASA-DLR Gravity Recovery and Climate Experiment (GRACE) mission. **Using laser ranging to LAGEOS-1 and LAGEOS-2, we demonstrate, with an error analysis based on several now-available GRACE gravity models, that the General Relativity prediction of the Lense-Thirring precession can be confirmed with an uncertainty better than 15%, in good agreement with previously published results.** In addition, with extensive modeling improvements in the various models, including the terrestrial reference frame and solid earth and ocean tides, we show that a credible experiment can be conducted with just four years of SLR overlapping the GRACE mission.

16th International Workshop on Laser Ranging, October 2008 Poznan, Poland



JEM03



EIGENGRACE02S

John’s office, Univ. Texas at Austin, nearly 20 years ago

EGM96 Model and its uncertainties

Even zonals **l m**	value	Uncer -tainty in value	Uncer- tainty on node I	Uncer- tainty on node II	
20	**- 0.48416 5 37 x 10-03**	**0.36x10-10**	**1 Ω_{LT}**	**2 Ω_{LT}**	
40	**0.53987 386 x 10-06**	**0.1 x 10-09**	**1.5 Ω_{LT}**	**0.5 Ω_{LT}**	
60	**- 0.14995 7 99 x 10-**	**0.15x10-09**	**0.6 Ω_{LT}**	**0.9 Ω_{LT}**	

3 main unknowns: δC_{20} ,δC_{40} and LT

Needed 3 observables

we only have 2: $\delta\Omega_{I}$,$\delta\Omega_{II}$

(orbital angular momentum vector)

EGM96 Model and its uncertainties

Even zonals **l m**	value	Uncer-tainty in value	Uncer-tainty on node I	Uncer-tainty on node II	Uncer-tainty on Perigee II
20	**-0.484165 37 x 10^{-03}**	**0.36×10^{-10}**	**1 Ω_{LT}**	**2 Ω_{LT}**	**0.8 ω_{LT}**
40	**0.53987386 x 10^{-06}**	**0.1×10^{-09}**	**1.5 Ω_{LT}**	**0.5 Ω_{LT}**	**2.1 ω_{LT}**
60	**-0.14995**	**0.15×10^{-09}**	**0.6 Ω_{LT}**	**0.9 Ω_{LT}**	**0.31 ω_{LT}**

3 main unknowns: $\delta C_{20}, \delta C_{40}$ and LT

Needed 3 observables:

2: $\delta\Omega_{I}, \delta\Omega_{II}$ (orbital angular momentum vector) plus

1: $\delta\omega_{II}$ **(Runge-Lenz vector)**

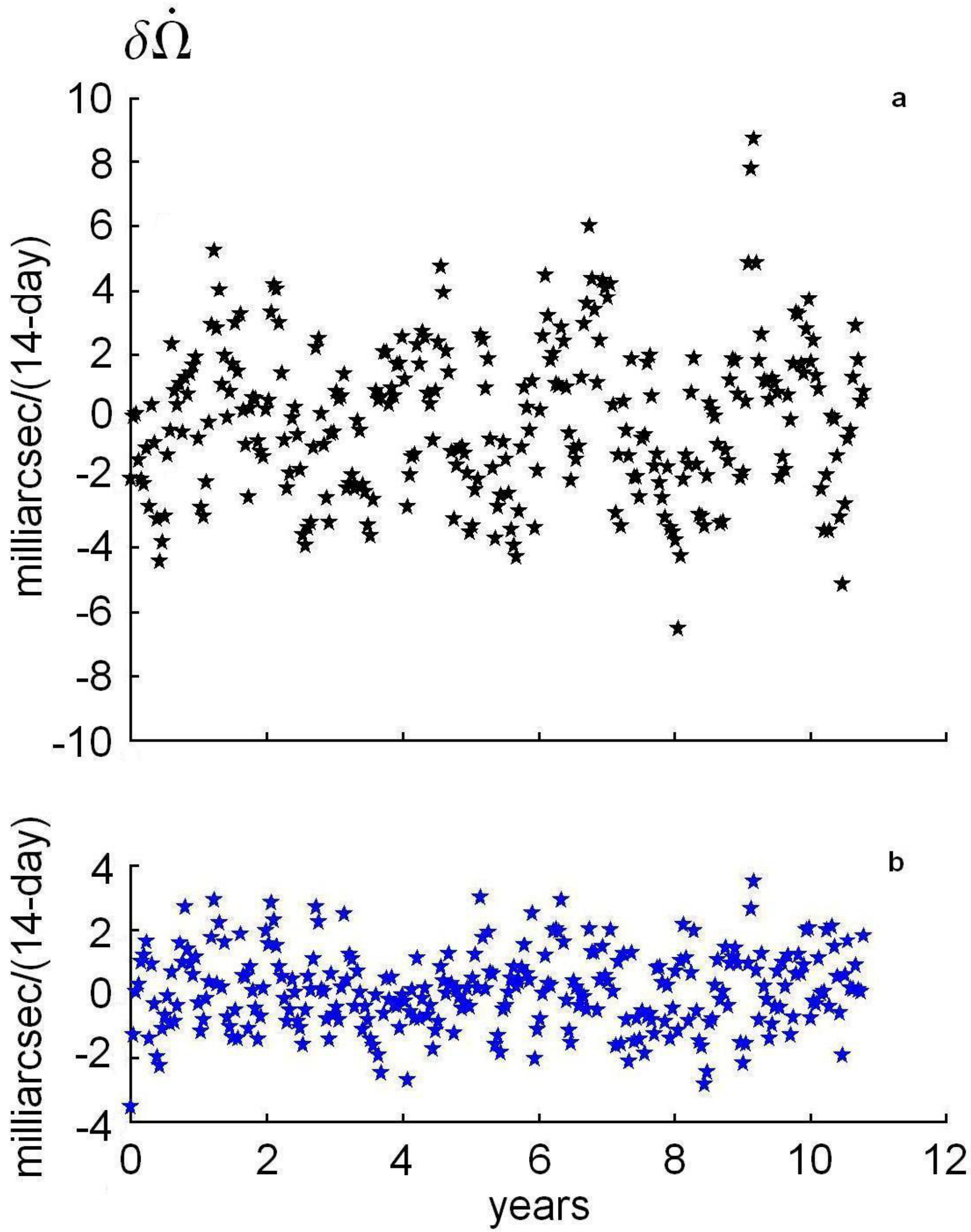
- **$\delta\Omega_{I} = K_2 \times \delta C_{20} + K_4 \times \delta C_{40} + K_{2n} \times \delta C_{2n,0} + \mu$ (31 mas/yr)**
- **$\delta\Omega_{II} = K'_2 \times \delta C_{20} + K'_4 \times \delta C_{40} + K'_{2n} \times \delta C_{2n,0} + \mu$ (31.5 mas/yr)**
- **$\delta\omega_{II} = K''_2 \times \delta C_{20} + K''_4 \times \delta C_{40} + K''_{2n} \times \delta C_{2n,0} - \mu$ (57 mas/yr)**

$$\mu = \delta\Omega_{I} + c_1\, \delta\Omega_{II} + c_2\, \delta\omega_{II}:$$

not dependent on δC_{20} and δC_{40}

(μ = 1 in GR)

I.C., PRL 1986; I.C., IJMP-A 1989; I.C., NC-A 1996



Post-fit residuals:
fit of linear trend only.

Post-fit residuals:
fit of linear trend plus
6 known frequencies

EIGEN-GRACE02S UNCERTAINTIES

GFZ POTSDAM EIGEN-GRACE02S GRACE-only gravity field

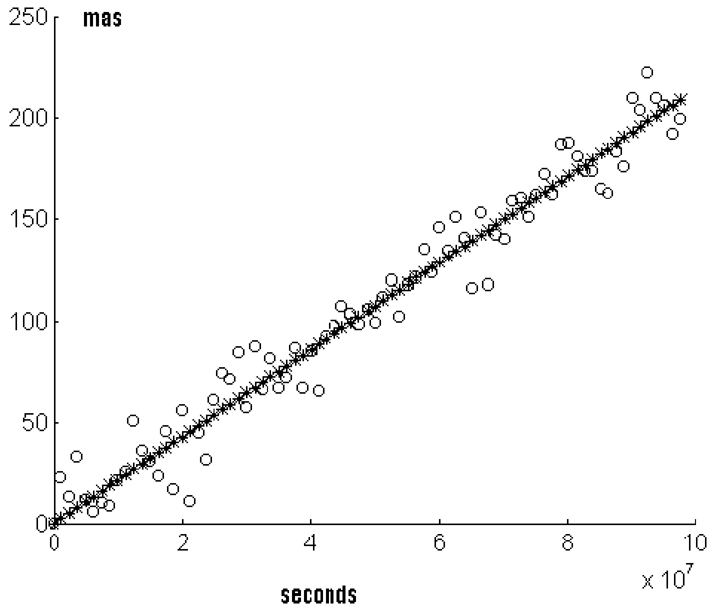
	l m	coefficient	formal error	error
GRCOF2	2 0	-.484165197888E-03	0.1433E-11	0.5304E-10
GRCOF2	4 0	0.539992946856E-06	0.4207E-12	0.3921E-11
GRCOF2	6 0	-.149930382378E-06	0.3037E-12	0.2049E-11
GRCOF2	8 0	0.494878910262E-07	0.2558E-12	0.1479E-11
GRCOF2	10 0	0.533212229998E-07	0.2347E-12	0.2101E-11
GRCOF2	12 0	0.364403114697E-07	0.2253E-12	0.1228E-11
GRCOF2	14 0	-.226739374086E-07	0.2233E-12	0.1202E-11
GRCOF2	16 0	-.470696292201E-08	0.2255E-12	0.9945E-12
GRCOF2	18 00	.609911161619E-08	0.2324E-12	0.9984E-12
GRCOF2	20 00	.215572707368E-07	0.2420E-12	0.1081E-11

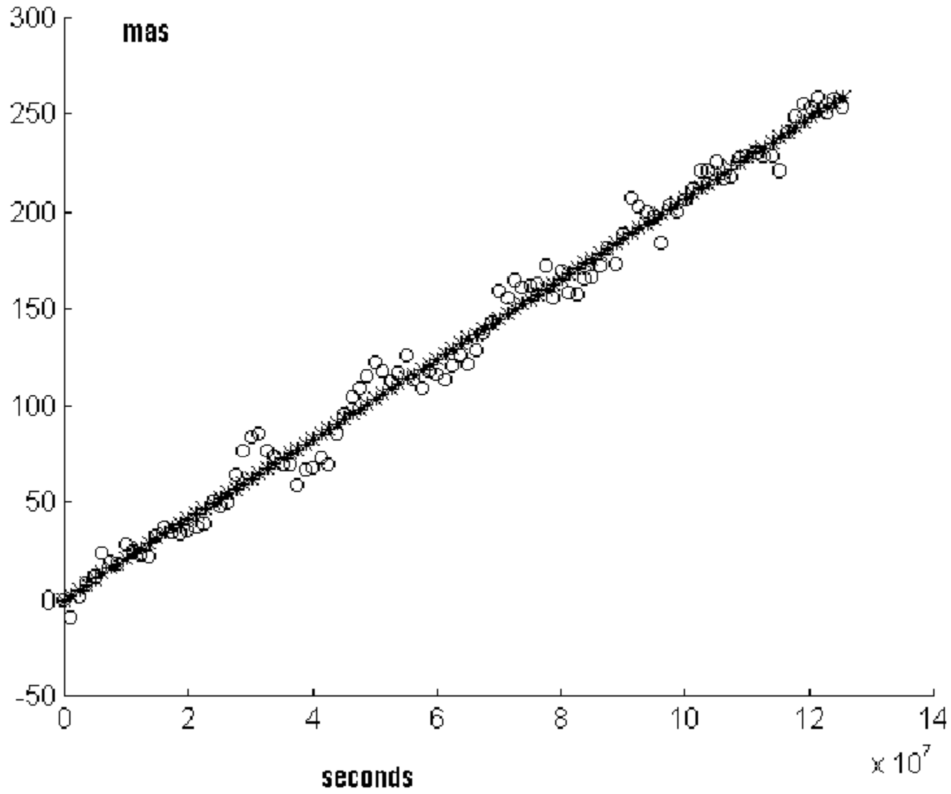

THE LARES

GRAVITY PROBE B

Problems with the GP-B data analysis have been outlined,
see, for example Prof. O'Connel :
http://www.phys.lsu.edu/faculty/oconnell/oconnell_pubs.html
(pub. number 307)

R. F. O'Connell, "Gravito-Magnetism in one-body and two-body systems: Theory and Experiment", in, "Atom Optics and Space Physics", Proc. of Course CLXVIII of the International School of Physics "Enrico Fermi", Varenna, Italy, 2007, ed. E. Arimondo, W. Ertmer and W. Schleich, to be published; and
G. Forst 2008: http://arxiv.org/PS_cache/arxiv/pdf/0712/0712.3934v1.pdf





I.C., et al. 1996-1997
(I.C. 1996). (Class.Q.Grav. ...)
Gravity model JGM-3
Obs. period 3.1 years
Result: $\mu \cong 1.1$

I.C., Pavlis et al. 1998 (Science)
I.C. 2000 (Class.Q.Grav.)
Gravity model EGM-96
Obs. period 4 years
Result: $\mu \cong 1.1$

IC

Nuovo Cimento A

1996

for LAGEOS II: $\dot{\omega}_{\mathrm{LAGEOS\,II}} \cong 160°/\mathrm{year}$, and the classical perigee precession is:

$$\dot{\omega}^{\mathrm{Class}} = -\frac{3}{4}\, n \left(\frac{R_\oplus}{a}\right)^2 \frac{1-5\cos^2 I}{(1-e^2)^2}\, J_2 - \tag{11}$$

$$-\left[\left[15 n R_\oplus^4 \left(108 + 135 e^2 + 208\cos(2I) + 252 e^2 \cos(2I) + 196\cos(4I) + 189 e^2 \cos(4I)\right)\right] / \left(1024 a^4 (1-e^2)^4\right)\right] J_4 + \Sigma P_{2n} \times J_{2n}\,,$$

where the P_{2n} are the coefficients (in the equation for the perigee rate) of the nonnormalized even zonal harmonics $J_{2n} \equiv -\sqrt{4n+1}\, C_{2n0}$. Thus, for the perigee of LAGEOS II, one has (in units of $\dot{\omega}_{\mathrm{II}}^{\mathrm{Lense\text{-}Thirring}}$):

	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to JGM3 estimated errors	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to difference (JGM3 – GEMT3)
δC_{20}	~ 1.1	~ 5.9
δC_{40}	~ 2.1	~ 5.3
δC_{60}	~ 0.41	~ 0.32
δC_{80}	~ 0.68	~ 0.8
$\delta C_{10,0}$	~ 0.22	~ 0.07

From these uncertainties in the perigee rate of LAGEOS II, similarly to what inferred for the nodal rates, it is manifest that the dominating error sources are due to the uncertainties in C_{20} and C_{40}.

Thus, summarizing, we have now the three unknowns δC_{20}, δC_{40} and Lense-Thirring effect, and the three observable quantities $\dot{\Omega}_{\mathrm{LAGEOS}}$, $\dot{\Omega}_{\mathrm{LAGEOS\,II}}$, and $\dot{\omega}_{\mathrm{LAGEOS\,II}}$.

The main unmodeled part of the LAGEOS I nodal rate, due to the uncertainties in the even zonal harmonics, to the errors in the value of the orbital parameters (mainly the inclination), and including the Lense-Thirring effect (to be determined), is:

$$\delta\dot{\Omega}_I = (-9.3\cdot 10^{11}) \times \delta C_{20} - (4.62\cdot 10^{11}) \times \delta C_{40} + \Sigma N_{2n} \times \delta C_{2n0} + 6 \times \delta I_I + 31\mu\,, \tag{12}$$

where $\delta\dot{\Omega}$ is in units of milliarcsec/year, and δI in milliarcsec. This formula shows the main error sources in the calculated nodal rate (apart from the errors due to tides and to nongravitational perturbations; see below). In this formula the first two contributions are due to the uncertainties δC_{20} and δC_{40}, we then have the error due to the uncertainties in the higher even zonal harmonics δC_{2n0} (with $2n \geqslant 6$), and the error due to the uncertainties in the determination of the inclination δI_I. In this formula we have also included the Lense-Thirring [2] parameter μ, by definition 1 in general relativity: $\mu^{\mathrm{GR}} \equiv 1$, that, if not incorporated in the modeling of the orbital perturbations, will affect the orbital residuals. One can write a similar expression for the node of LAGEOS II:

$$\delta\dot{\Omega}_{\mathrm{II}} = (17.17\cdot 10^{11}) \times \delta C_{20} + (1.68\cdot 10^{11}) \times \delta C_{40} + \Sigma N''_{2n} \times \delta C_{2n0} + 5.3 \times \delta I_{\mathrm{II}} + 31.5\mu \tag{13}$$

CONCEPT OF THE LAGEOS III EXPERIMENT