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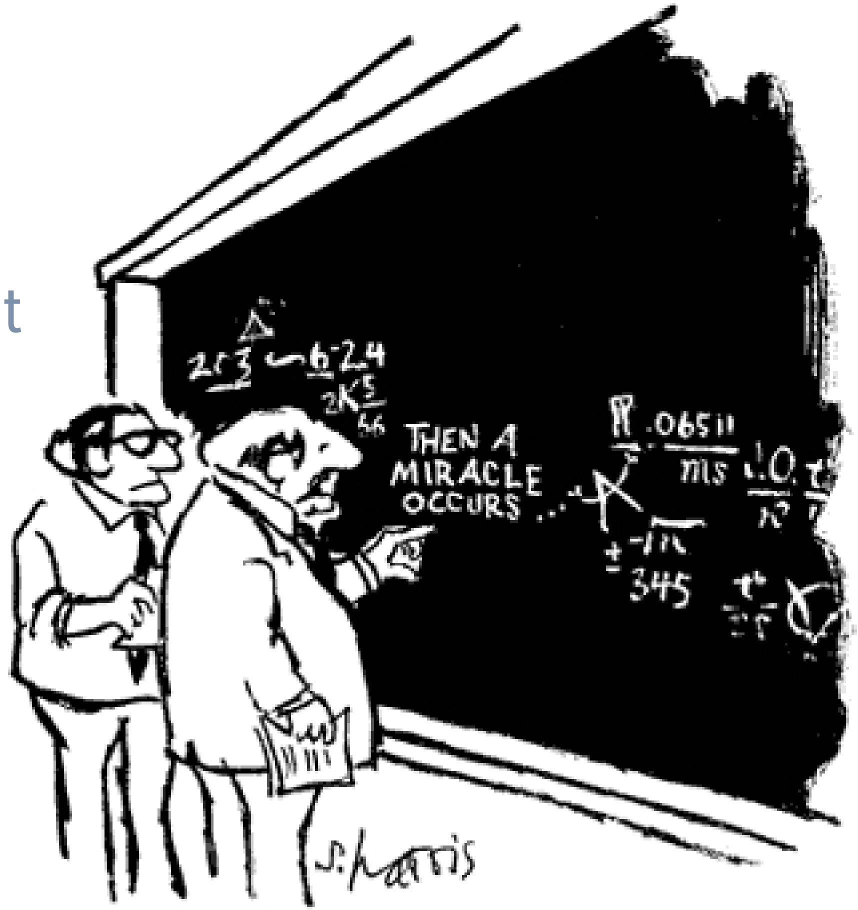
Sagnac effect in general relativity and Gödel's Universe

Endre Kajari, Wolfgang P. Schleich, First LARES workshop,
Sapienza, Universita di Roma, July 4th, 2009

Outline of the talk:

- Gödel's Universe
- Sagnac's original experiment and its description in GR
- Sagnac effect in a proper reference frame (PRF)
- Double eight-loop interferometer (DELI)



"I think you should be more explicit here in step two."

S. Harris

Why did Gödel bother himself with rotating universes?

- In Sept. 1946 Einstein received a letter by Gamov
- Einsteins response: "What does it mean, that the Universe as a whole possesses an angular momentum?"
- It is very likely that Einstein discussed the suggestion of Gamov with Gödel
- May 7th, 1949: Gödel's lecture on rotating universes at the Institute for Advanced Studies

Gödel's Universe

Line element in Gödel's Universe

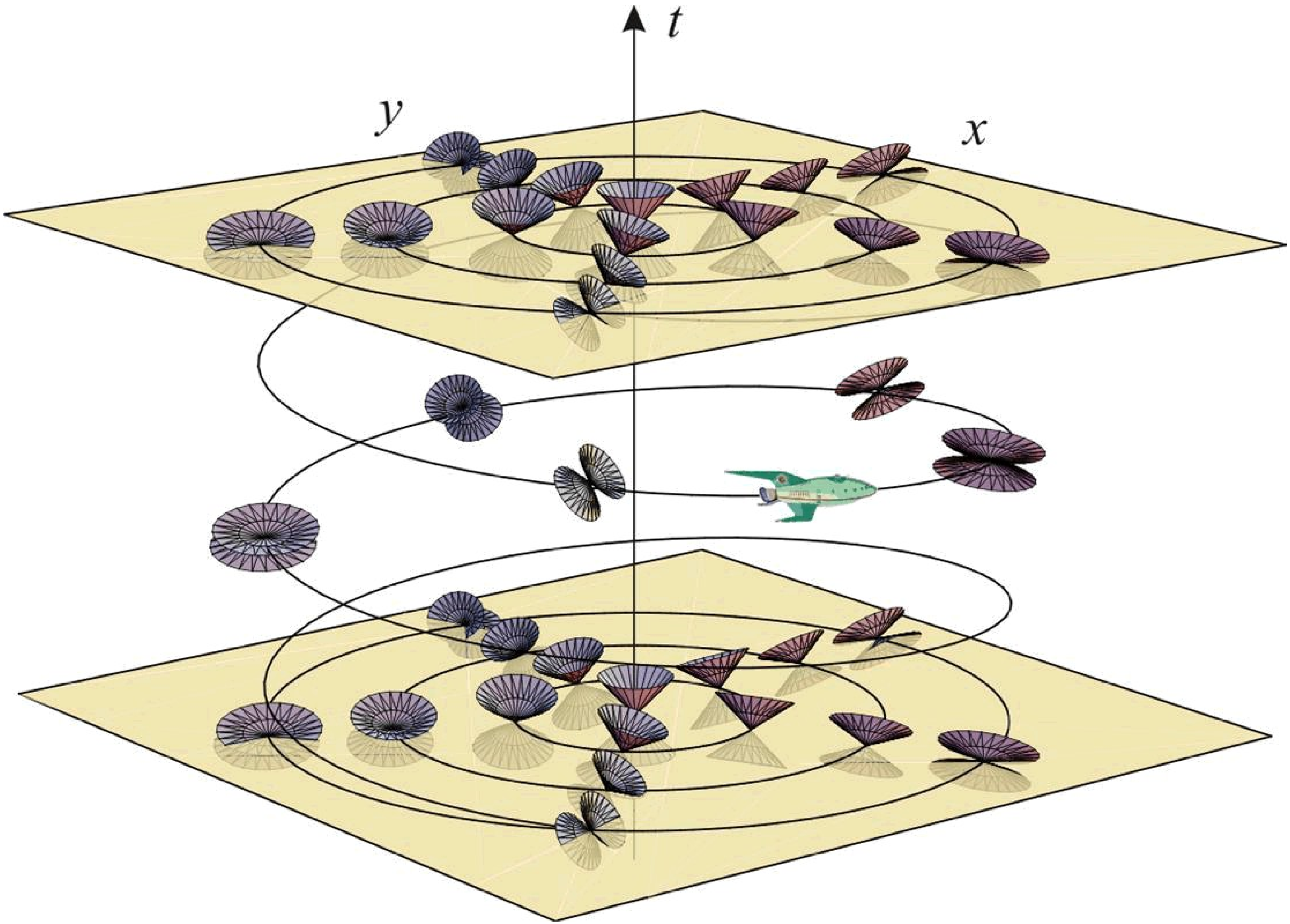
$$ds^2 = c^2dt^2 - \frac{dr^2}{1+\left(\frac{r}{2a}\right)^2} - r^2\left(1-\left(\frac{r}{2a}\right)^2\right)d\phi^2 - dz^2 + 2r^2\frac{c}{\sqrt{2}a}dtd\phi$$

In the limit of large critical Gödel radius: flat spacetime

$$ds^2 \xrightarrow{a\to\infty} c^2dt^2 - dr^2 - r^2d\phi^2 - dz^2$$

Gödel's Universe is (i) spacetime homogeneous
(ii) stationary
(iii) anisotropic
(iv) rotational symmetric with a constant rotation vector along z-axis

Closed Timelike Worldlines in Gödel's Universe



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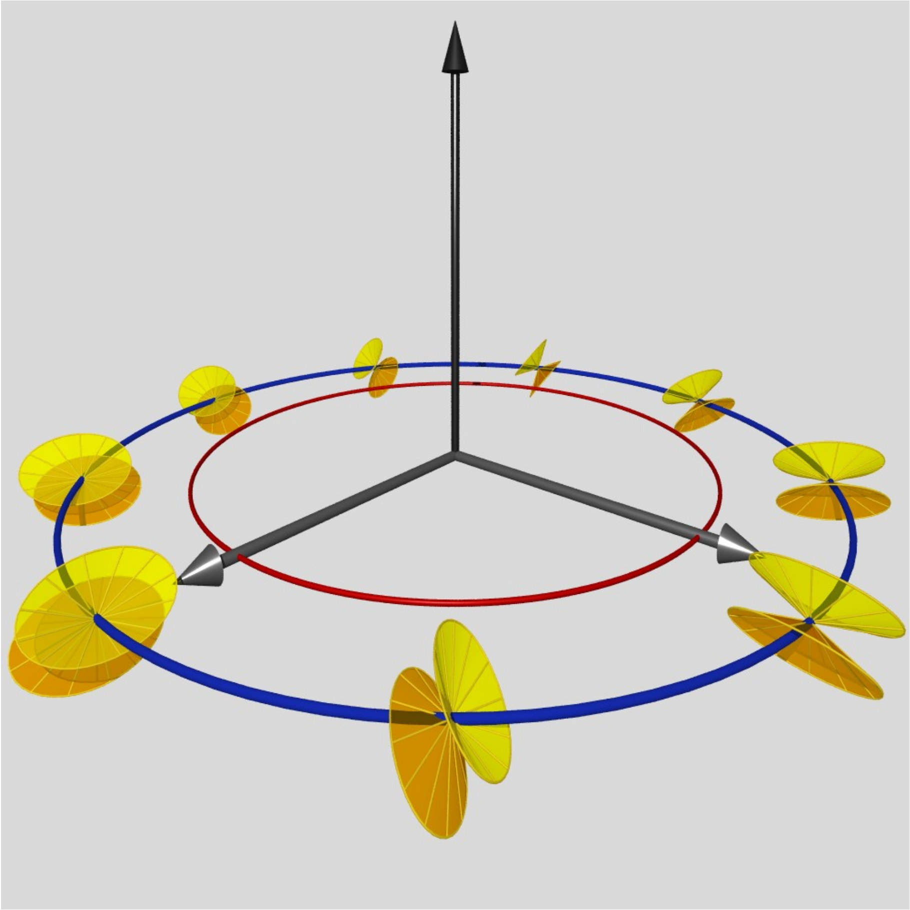
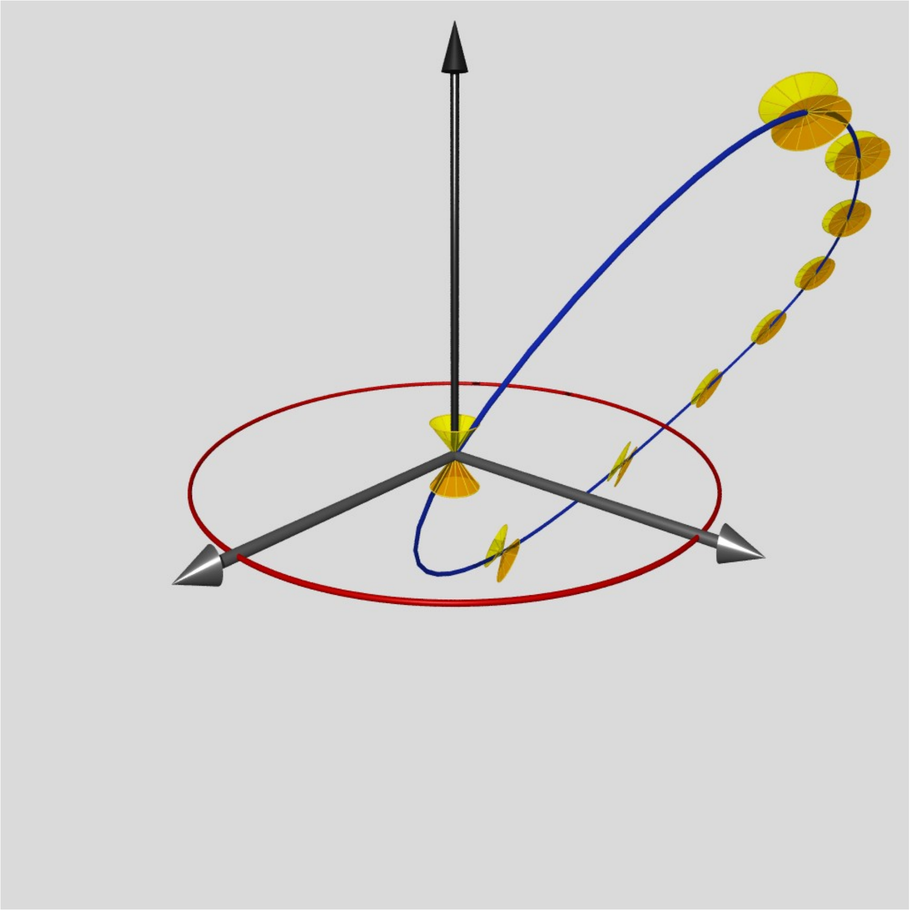
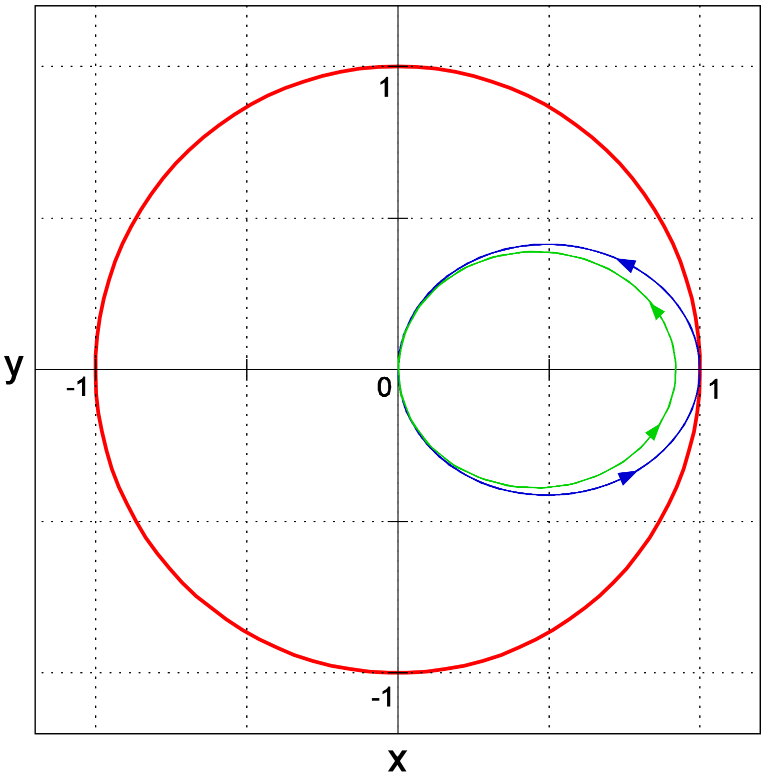
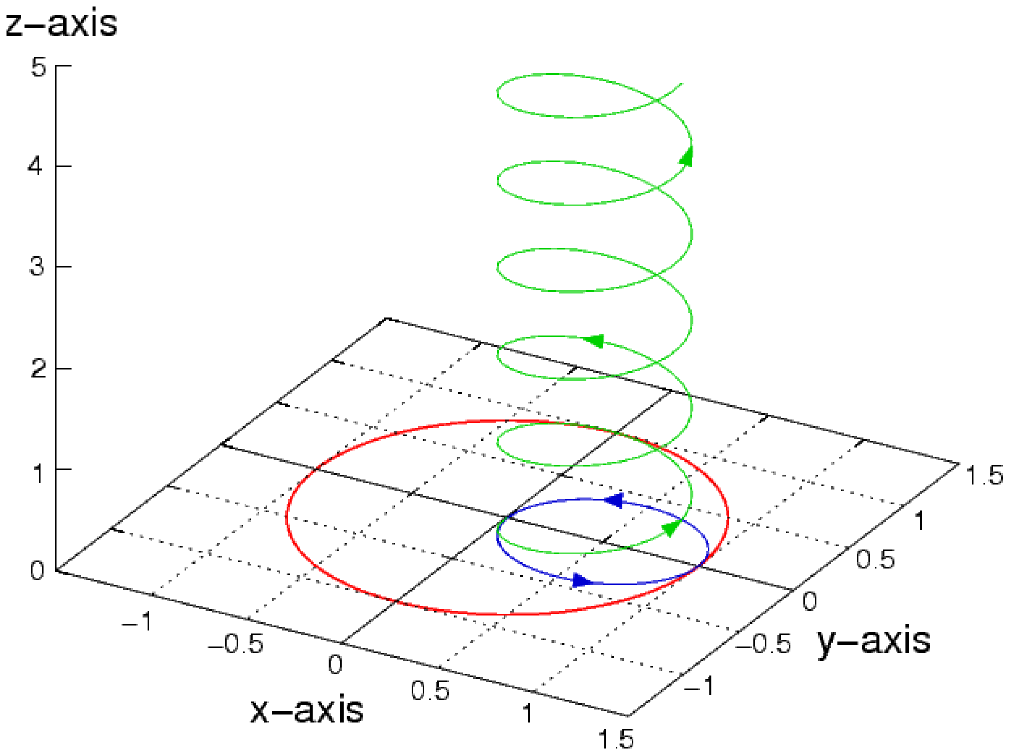




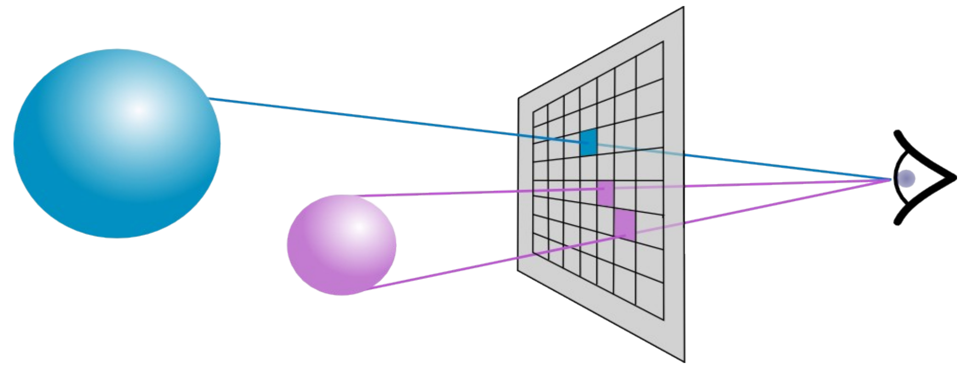
Null-geodesics starting at the origin



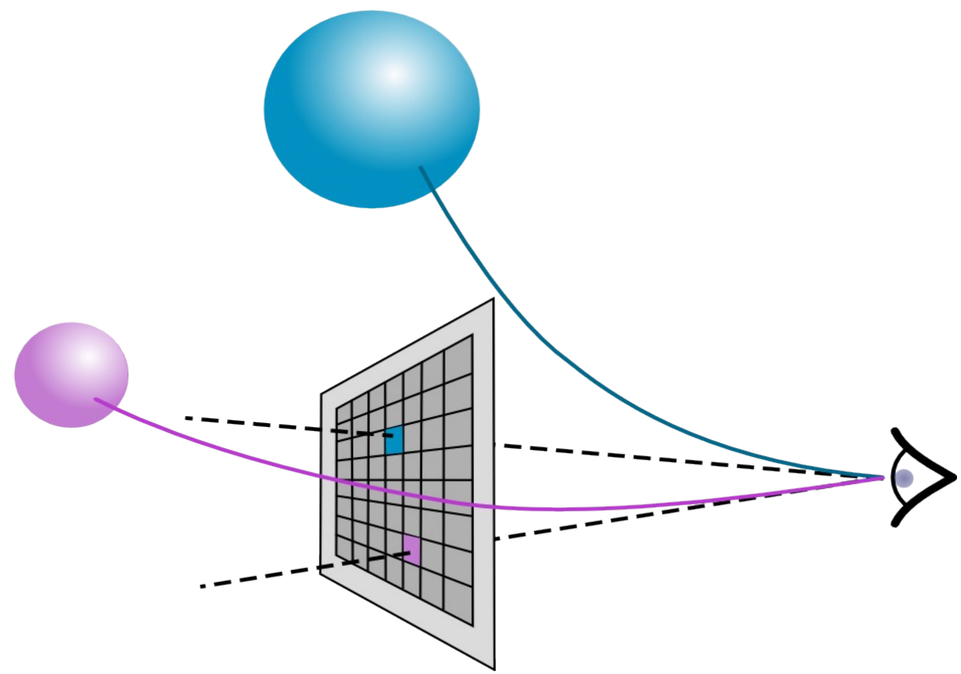


Visualization in General Relativity

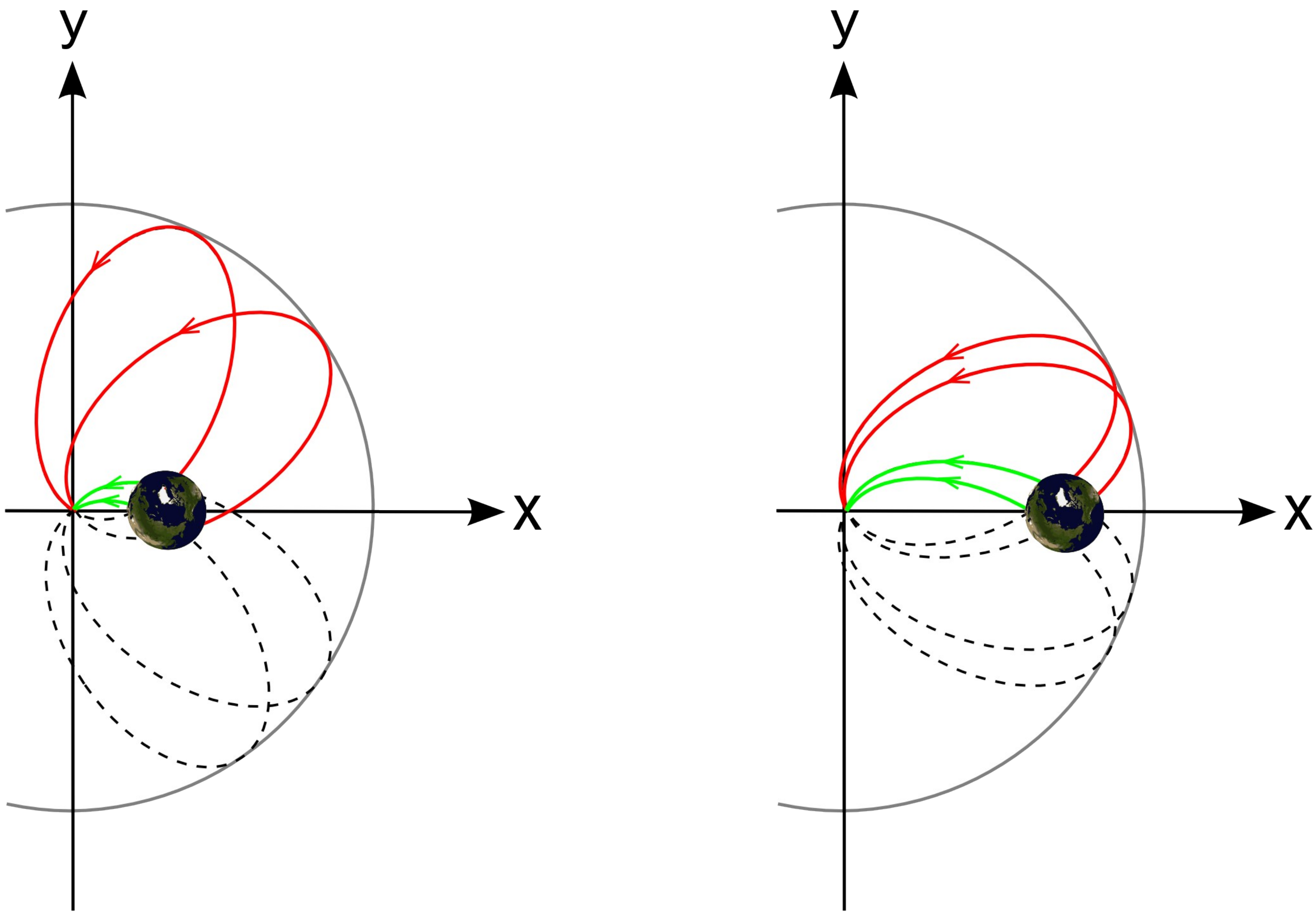
Flat spacetime



Curved spacetime



View on Earth in Gödel's Universe



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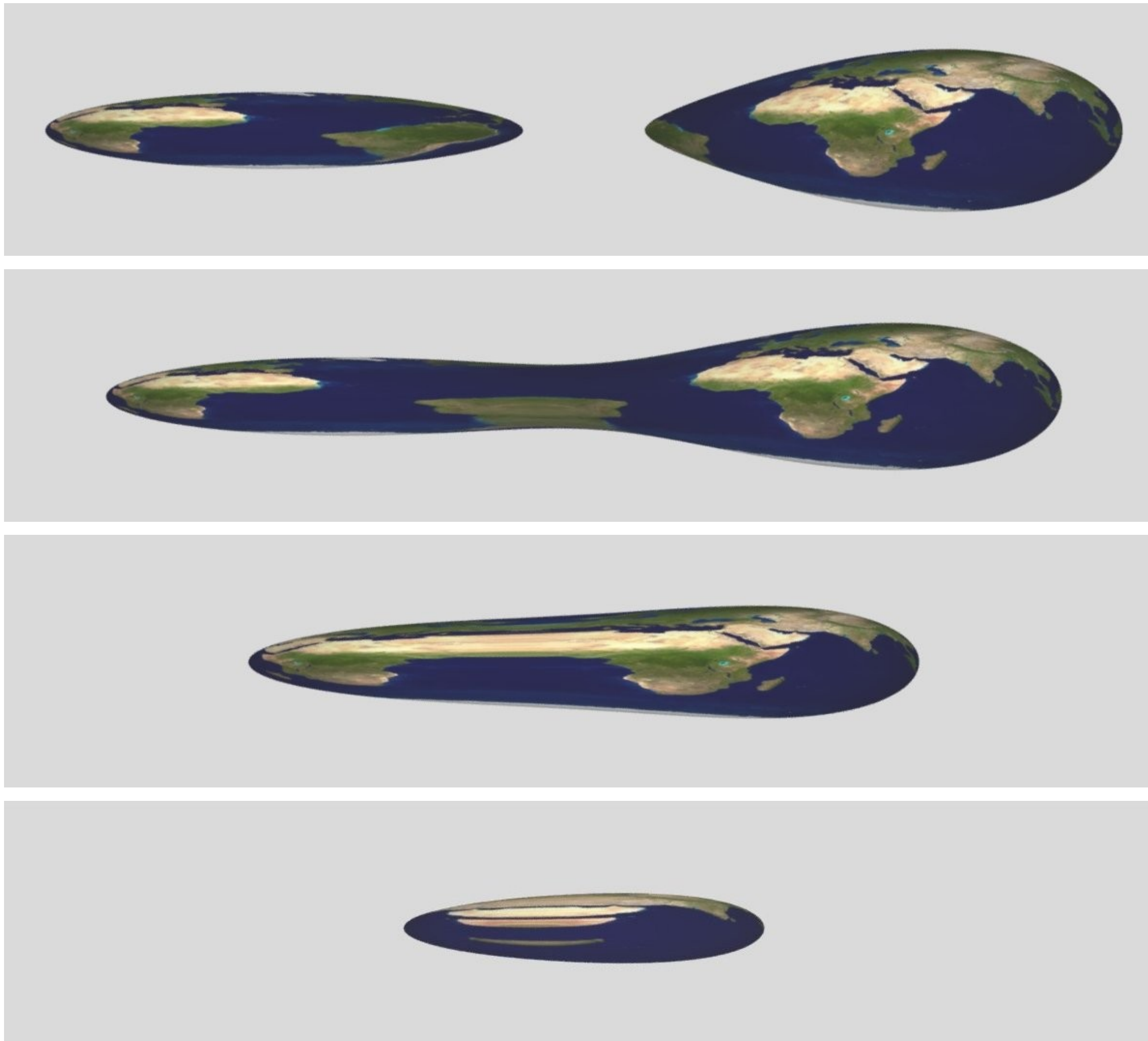

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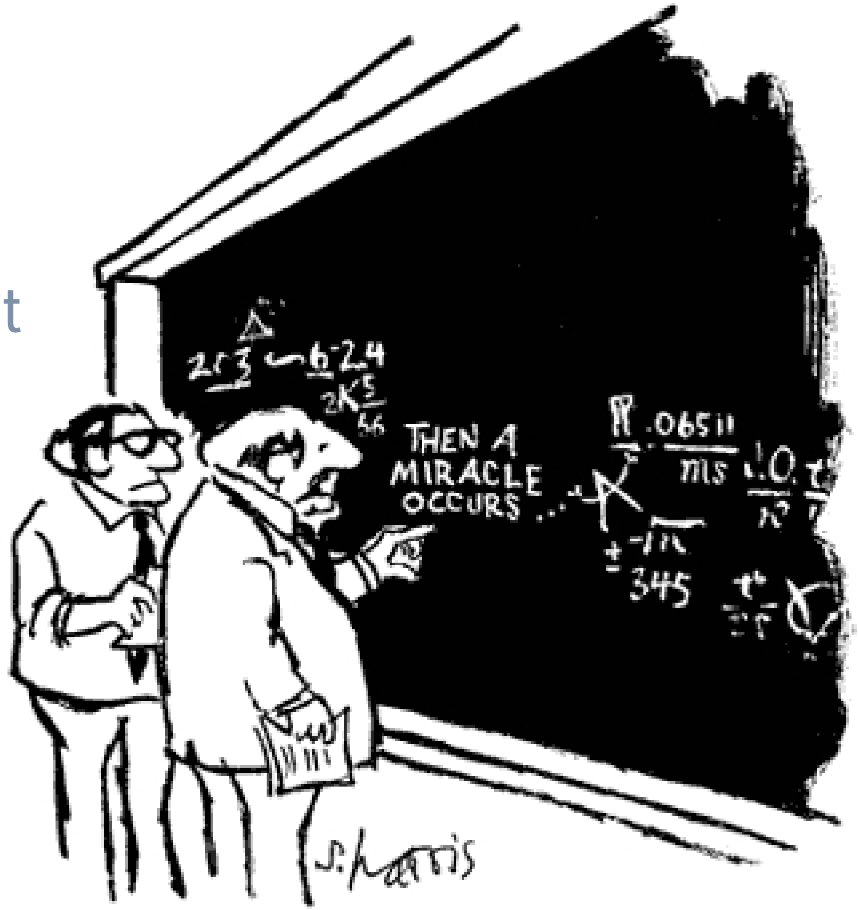

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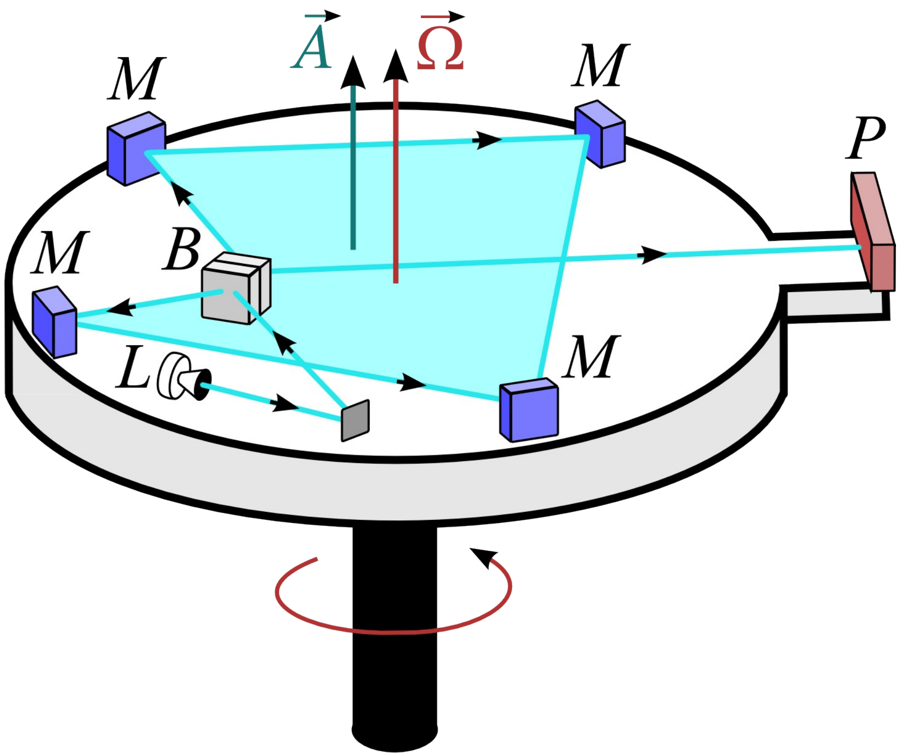
Sagnac's Original Experiment



$$\Delta t = \frac{4}{c^2}\vec{A}\cdot\vec{\Omega}$$

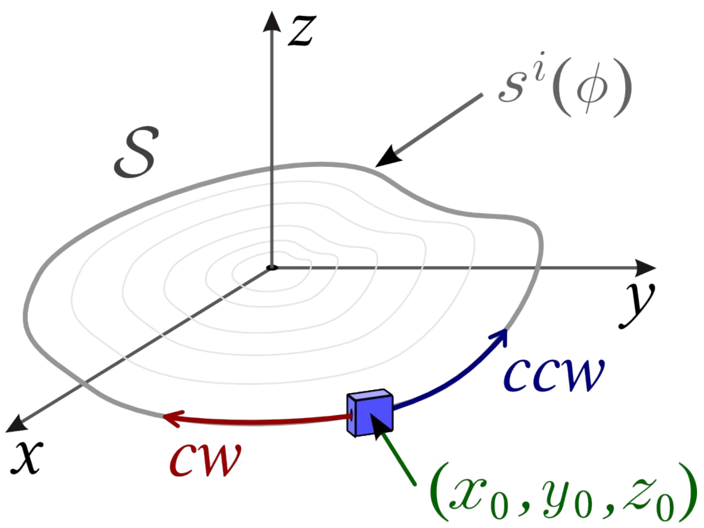
Sagnac's Conclusion:

"The observed interference effect is clearly the optical whirling effect due to the movement of the system in relation to the ether and directly manifests the existence of the ether, supporting necessarily the light waves of Huygens and of Fresnel."

C. R. Acad. Sci. **157**, 708 and 1410 (1913), translated by R. Hazelett

Sagnac Time Delay in General Relativity

for a time independent metric



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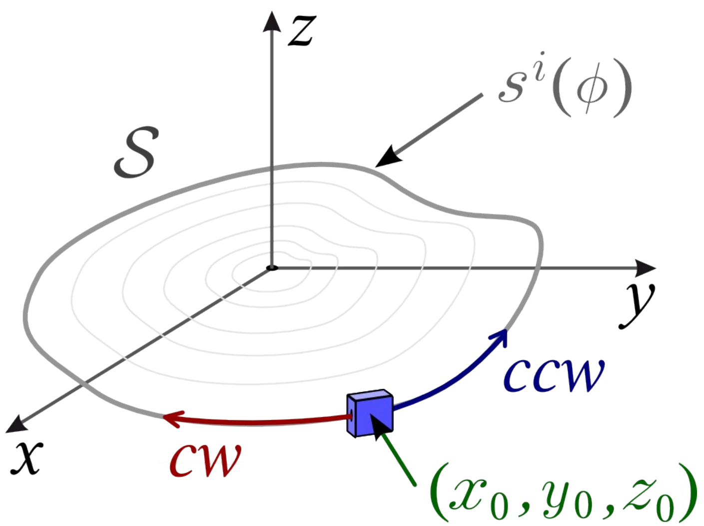

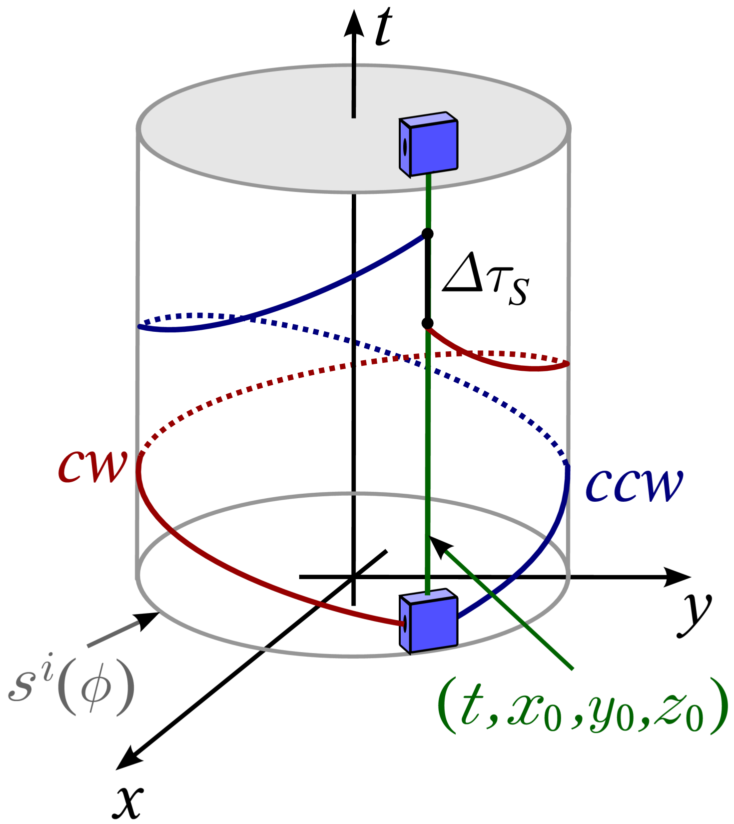

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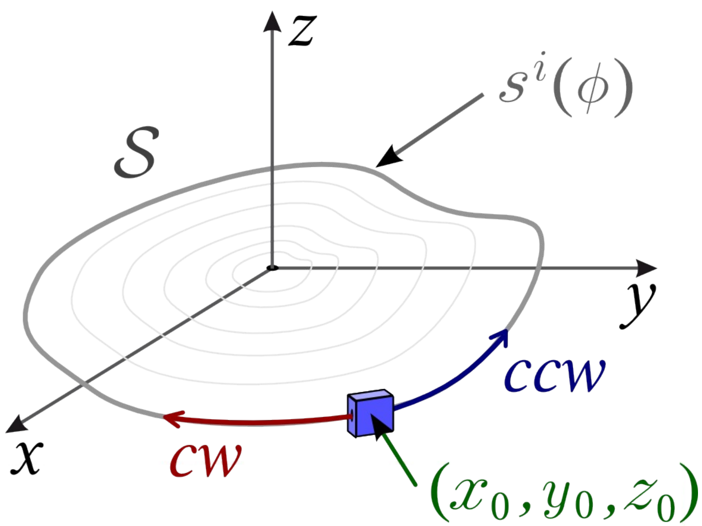
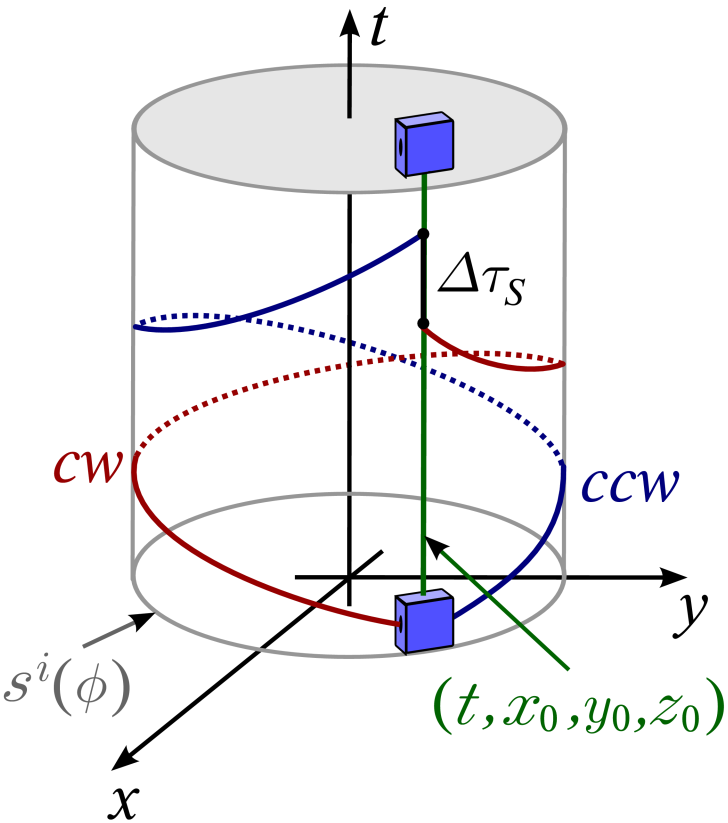
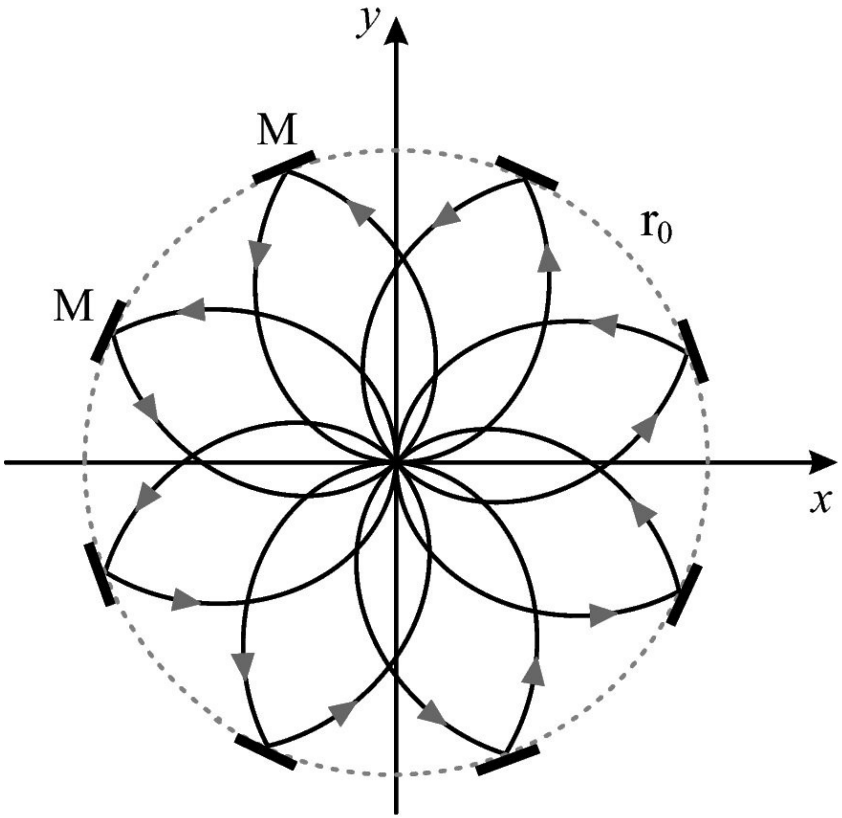
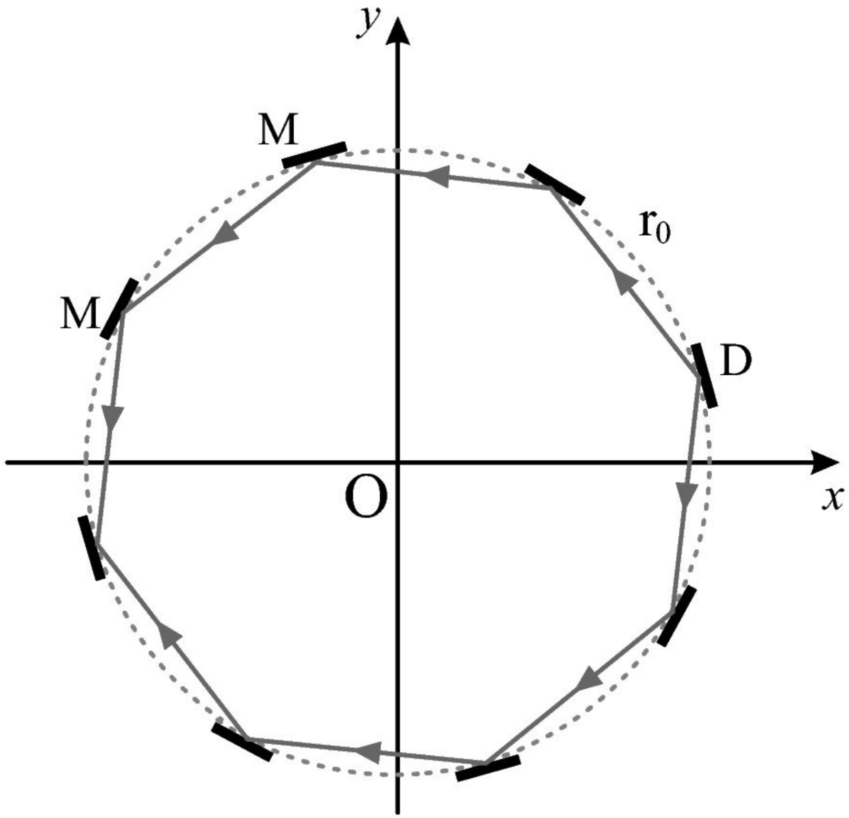




$$\Delta\tau_S = -\frac{2}{c}\sqrt{g_{00}(q^r)}\oint_{\mathcal{S}} \frac{g_{0i}}{g_{00}}\, ds^i$$

Motivation





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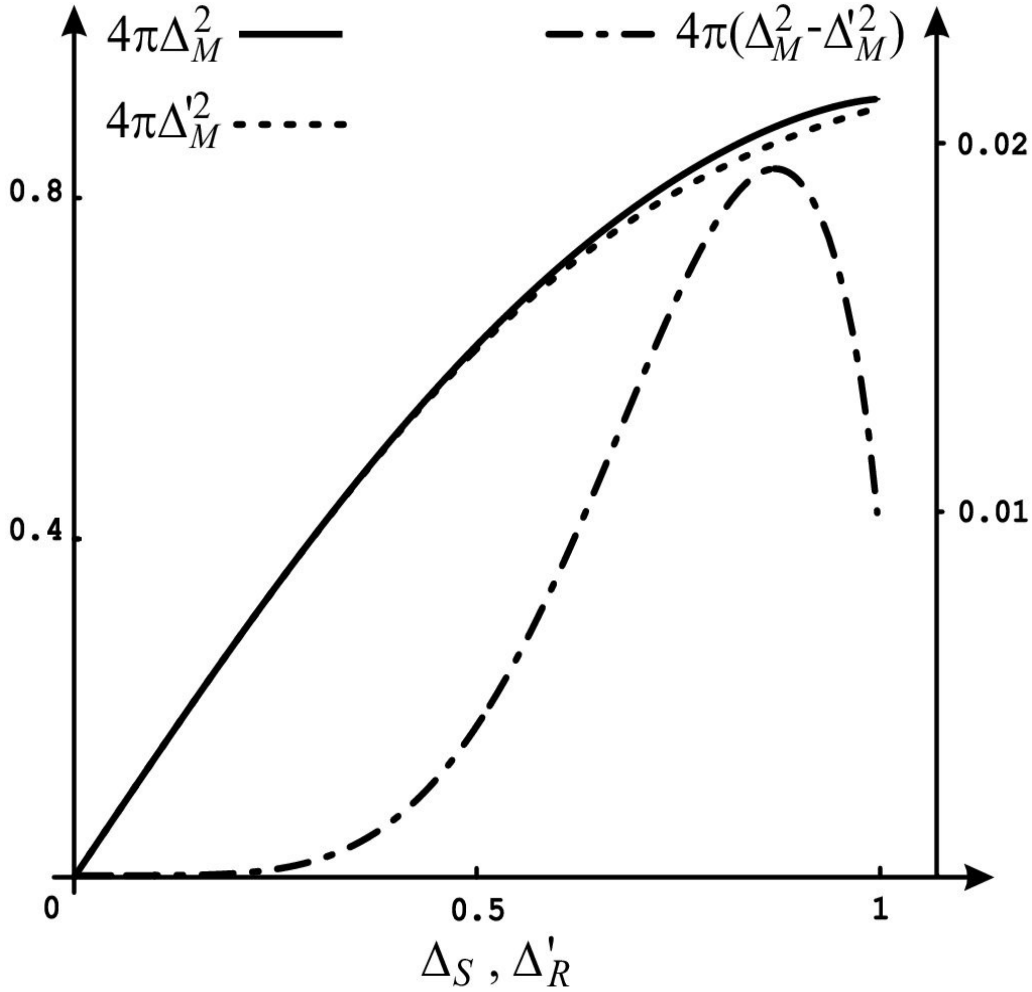

$4\pi\Delta_M^2$
$4\pi\Delta_M'^2$
$4\pi(\Delta_M^2 - \Delta_M'^2)$
0.8
0.4
0.02
0.01
0
0.5
1
Δ_S , Δ_R'

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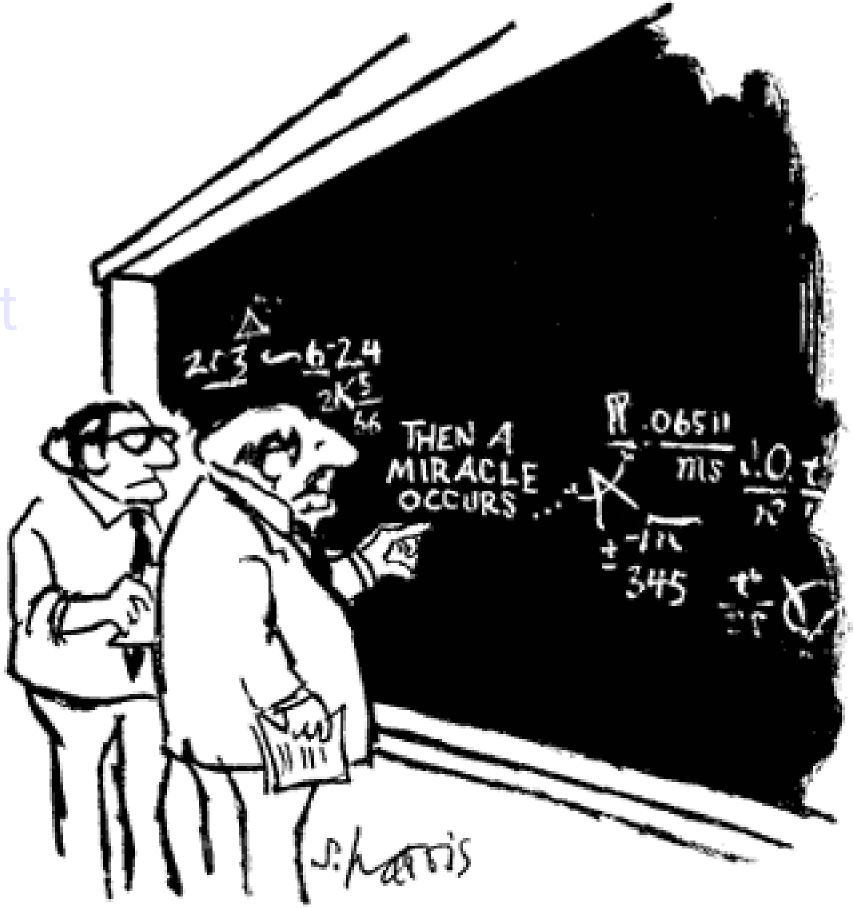

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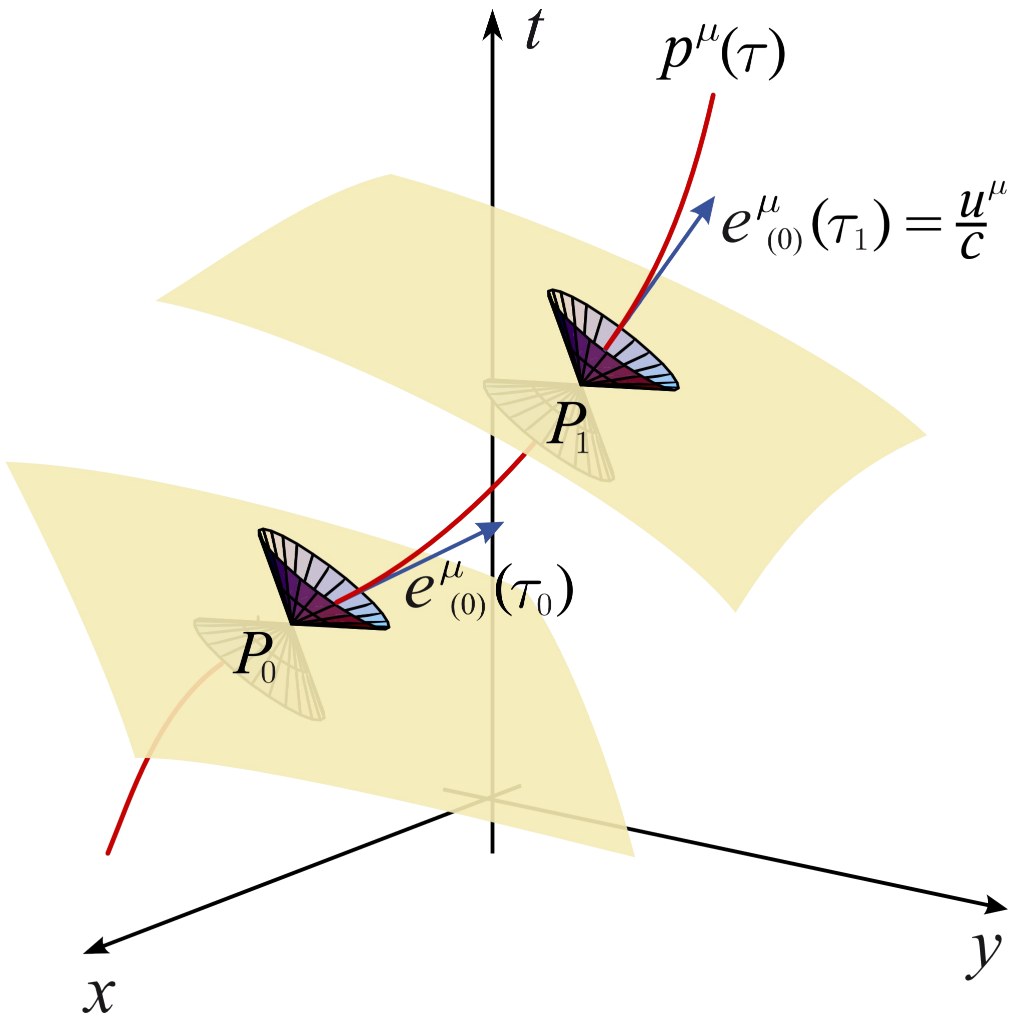
Proper Reference Frame (PRF)



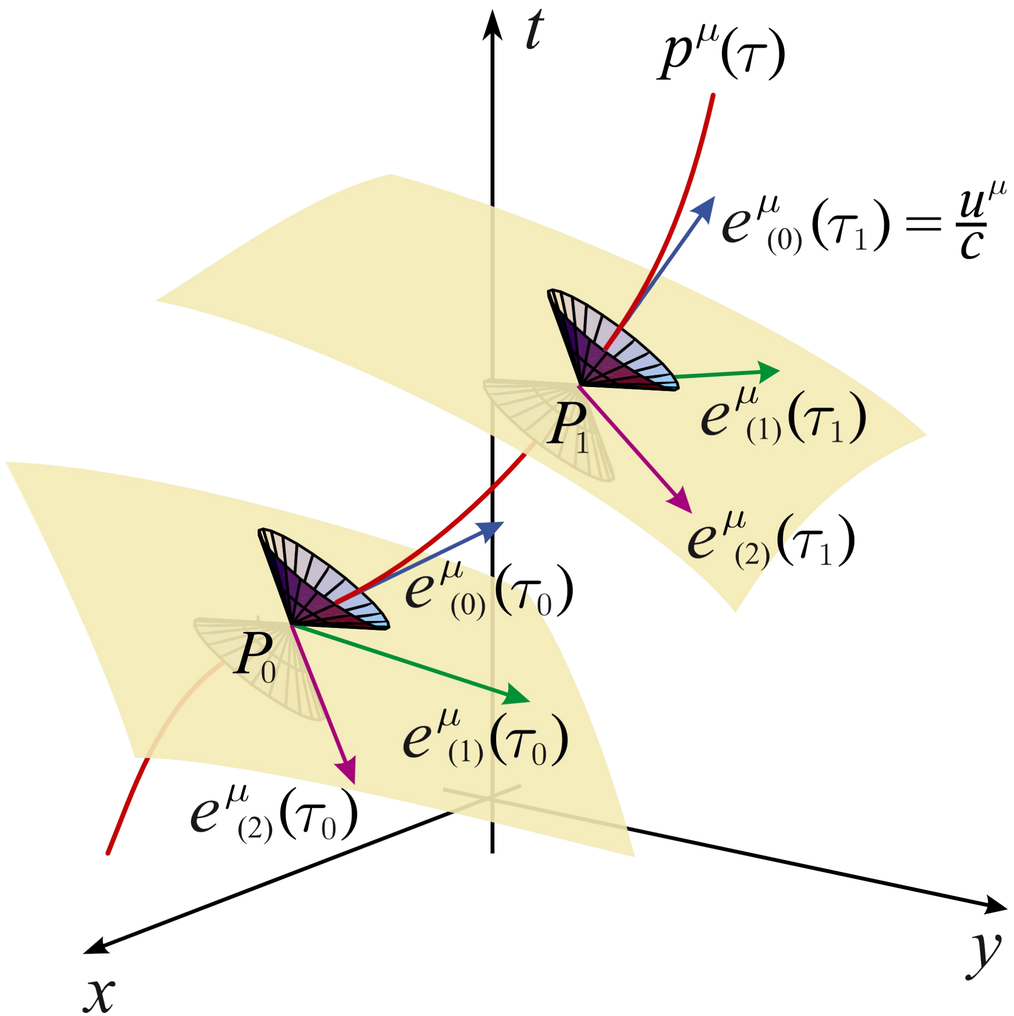
four-velocity:

$$u^\mu(\tau) = \frac{dp^\mu}{d\tau}$$

four-acceleration:

$$a^\mu(\tau) = u^\mu{}_{;\nu}\, u^\nu$$

Proper Reference Frame (PRF)



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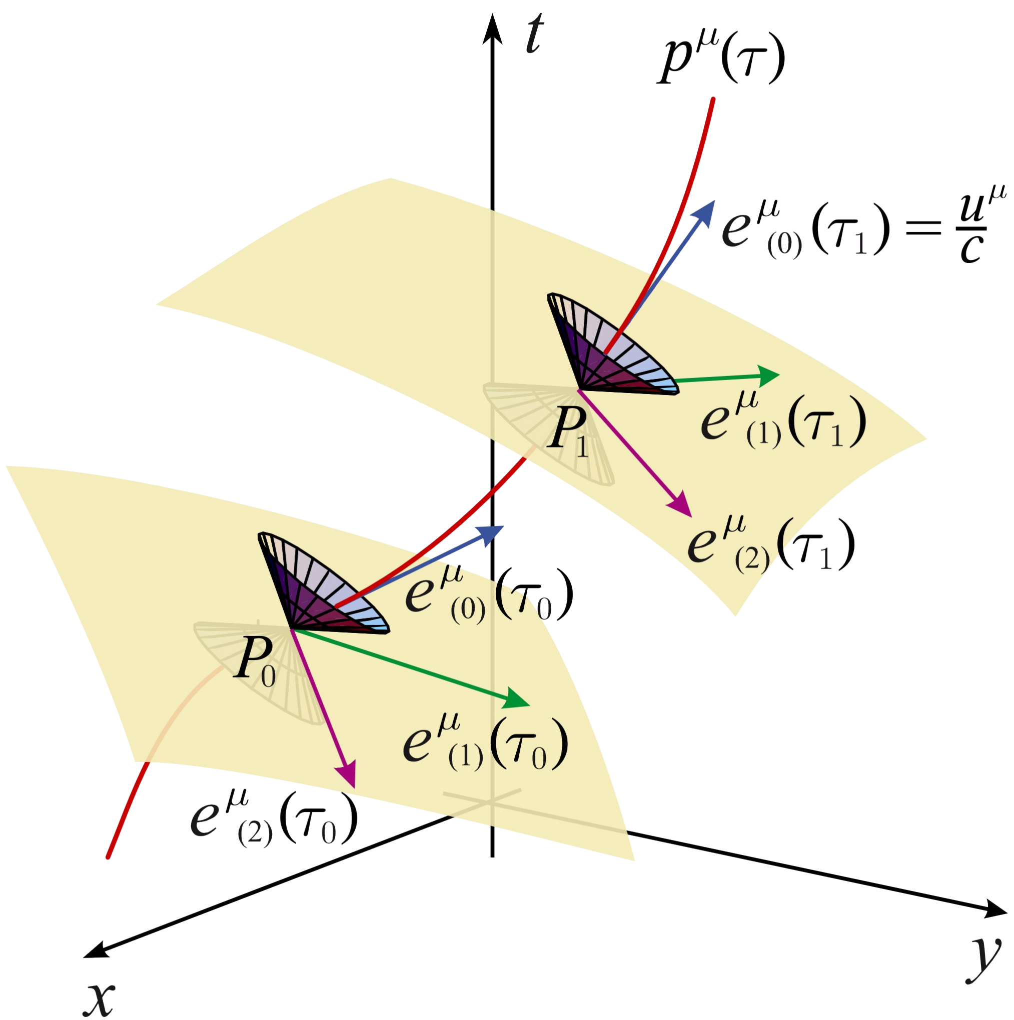
orthonormal tetrads:

$$e^\mu_{(\alpha)}(\tau)\, e^\nu_{(\beta)}(\tau)\, g_{\mu\nu}(p^\sigma(\tau)) = \eta_{(\alpha\beta)}$$

Proper Reference Frame (PRF)



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orthonormal tetrads:

$$e^\mu{}_{(\alpha)}(\tau)\, e^\nu{}_{(\beta)}(\tau)\, g_{\mu\nu}(p^\sigma(\tau)) = \eta_{(\alpha\beta)}$$

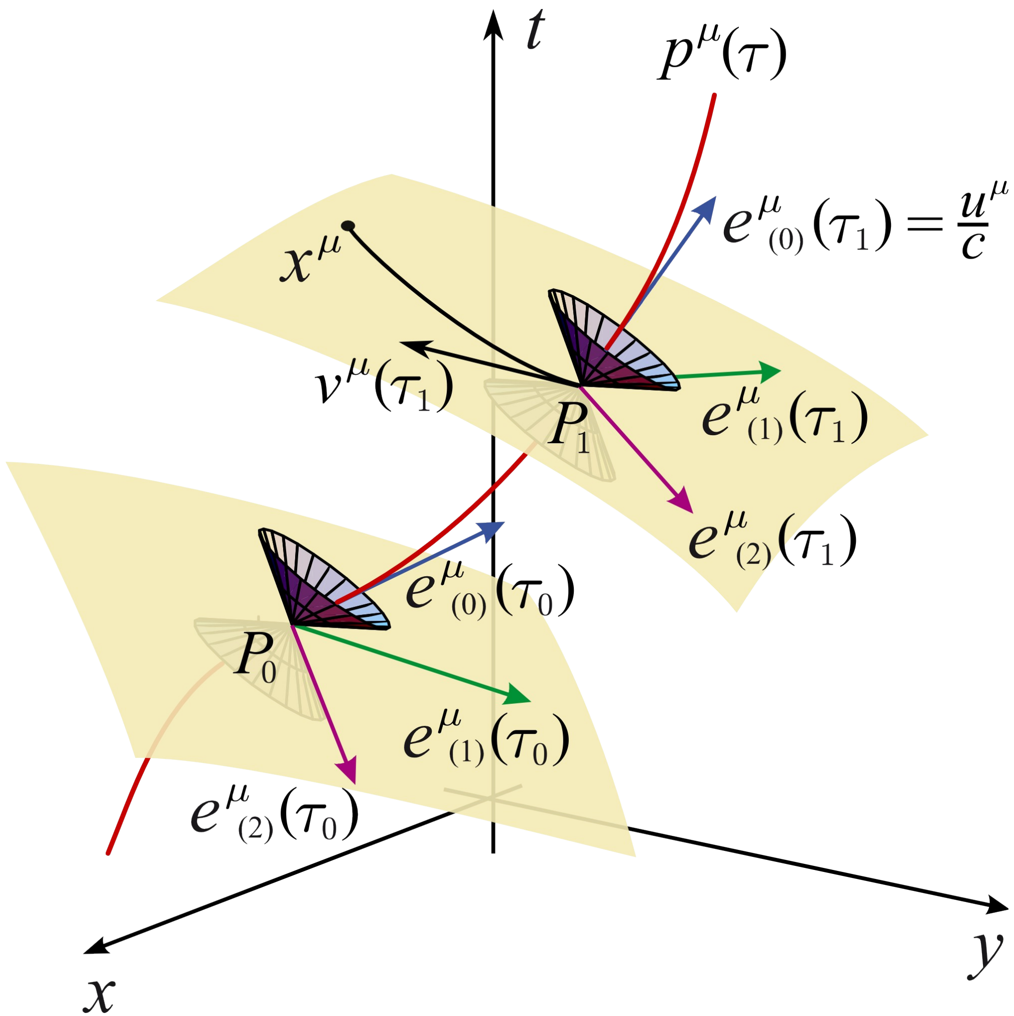
proper transport:

$$e^\mu{}_{(\alpha);\nu}\, u^\nu = -\Omega^\mu{}_\nu\, e^\nu{}_{(\alpha)}$$

transport matrix:

$$\Omega^{\mu\nu} = -\frac{1}{c^2}\left(a^\mu u^\nu - a^\nu u^\mu\right) + \frac{1}{c}\, u_\rho\, \omega_\sigma\, \varepsilon^{\rho\sigma\mu\nu}$$

Proper Reference Frame (PRF)



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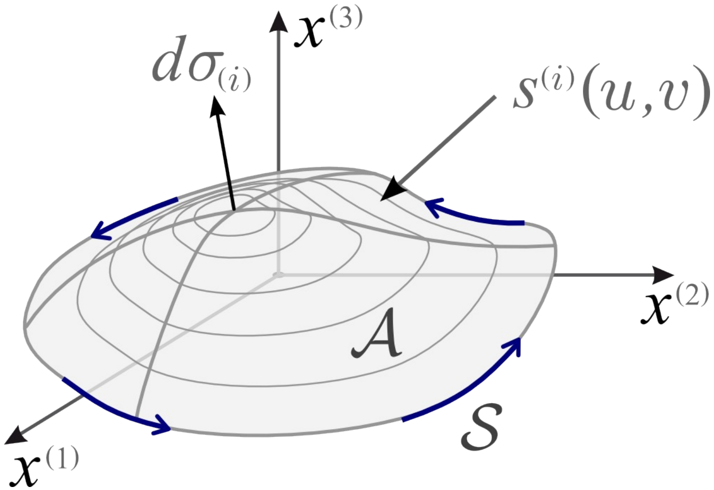
Sagnac Time Delay in a PRF

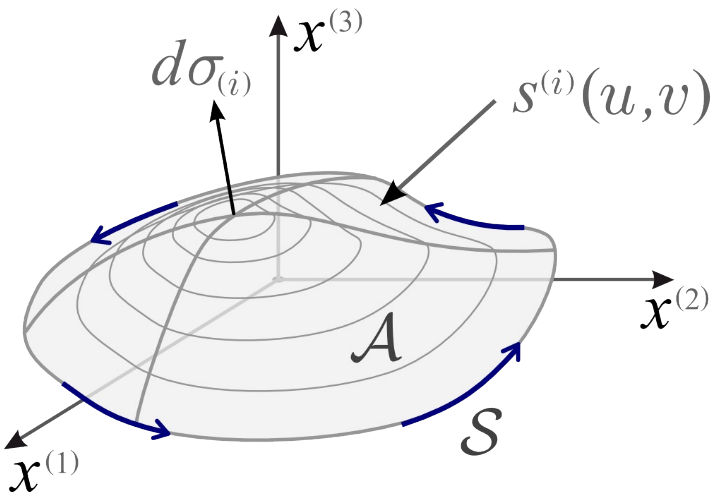


general surface

$$A_{(a)} = \iint_{\mathcal{A}} d\sigma_{(a)}$$

$$A_{(a)}^{(i_1)} = \iint_{\mathcal{A}} s^{(i_1)} d\sigma_{(a)}$$

Sagnac Time Delay in a PRF



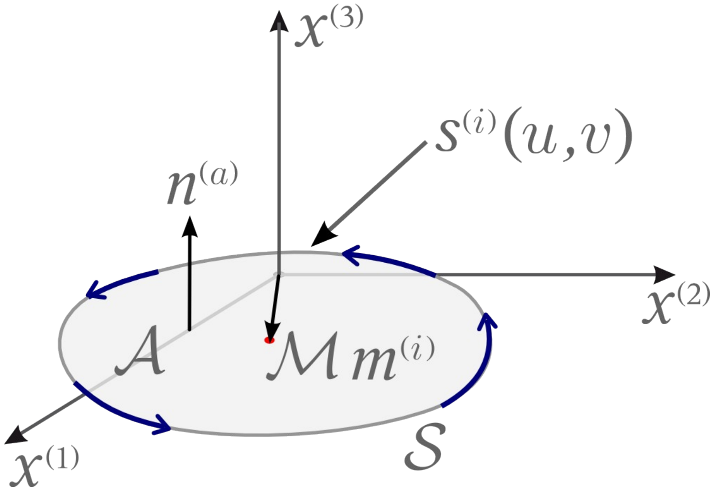
general surface

$$A_{(a)} = \iint_{\mathcal{A}} d\sigma_{(a)}$$

$$A_{(a)}^{(i_1)} = \iint_{\mathcal{A}} s^{(i_1)} d\sigma_{(a)}$$

$$\Delta\tau_S = \frac{4}{c^2}\sqrt{g_{(00)}(q^{(r)})}\Big[-\omega^{(a)}A_{(a)} + \frac{2c}{3}\varepsilon^{(0ajk)}R_{(0)\{(i_1)(j)\}(k)}(p^{(r)})\,A_{(a)}^{(i_1)} + \frac{1}{c^2}\left(\omega^{(l)}a_{(l)}\,\delta_{(i_1)}^{(a)} - 3\,\omega^{(a)}a_{(i_1)}\right)A_{(a)}^{(i_1)} + \mathcal{O}\left(A_{(a)}^{(i_1)(i_2)}\right)\Big]$$

Sagnac Time Delay in a PRF



planar surface

$$A_{(a)} = \mathcal{A}\, n_{(a)}$$

$$A_{(a)}^{(i_1)} = \mathcal{M}\, m^{(i_1)}\, n_{(a)}$$

$$\Delta\tau_S = \frac{4}{c^2}\sqrt{g_{(00)}(q^{(r)})}\,\Big[-\omega^{(a)} A_{(a)} + \frac{2c}{3}\,\varepsilon^{(0ajk)}\, R_{(0)\{(i_1)(j)\}(k)}(p^{(r)})\, A_{(a)}^{(i_1)}$$
$$+ \frac{1}{c^2}\left(\omega^{(l)} a_{(l)}\, \delta_{(i_1)}^{(a)} - 3\,\omega^{(a)} a_{(i_1)}\right) A_{(a)}^{(i_1)} + \mathcal{O}\Big(A_{(a)}^{(i_1)(i_2)}\Big)\Big]$$

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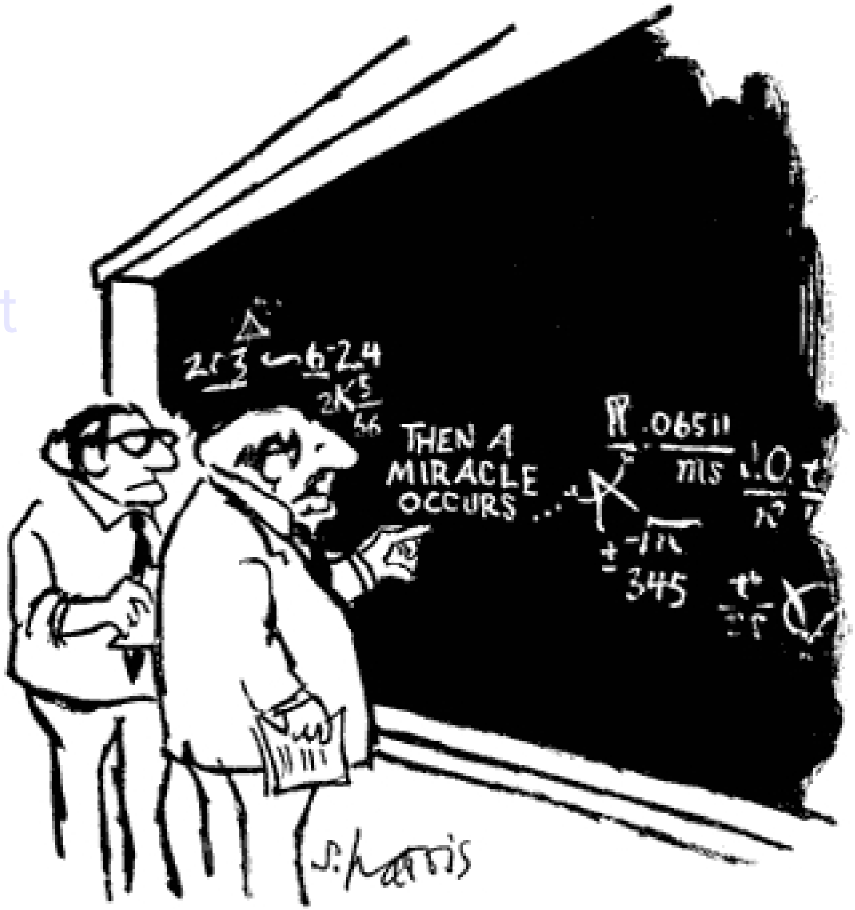

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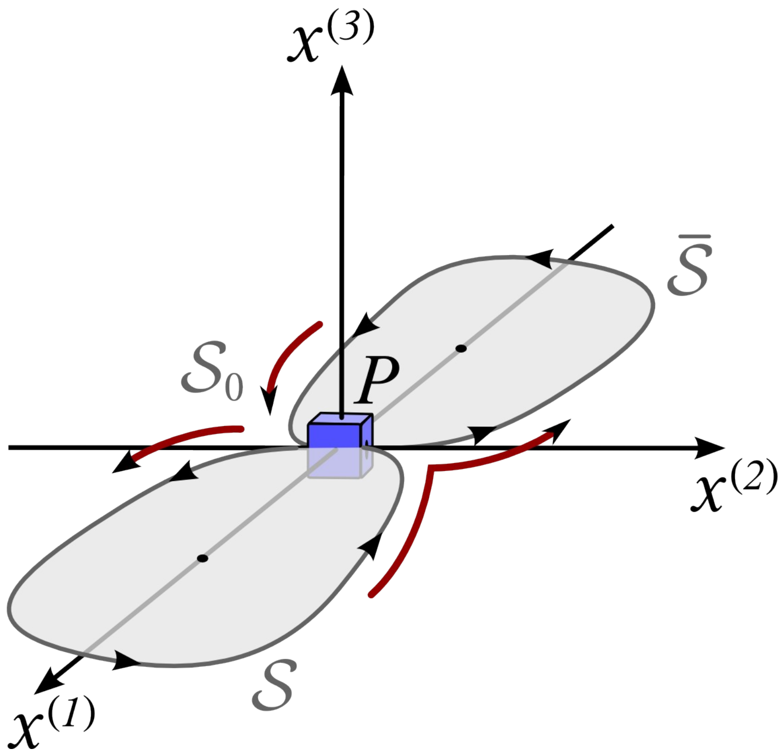
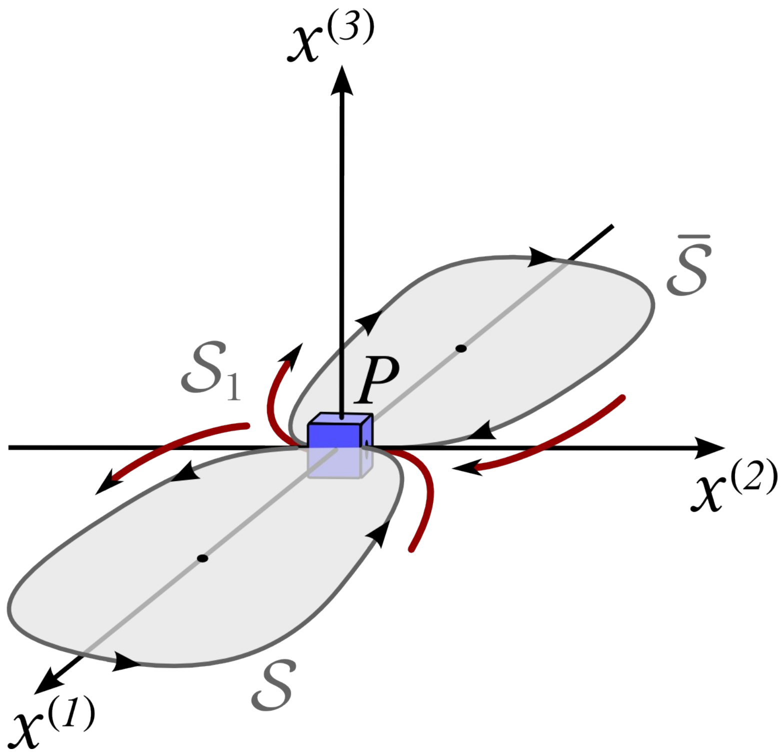
DELI: Measurement mode 1



$$\Delta\tau_{Sp}(\mathcal{S}_0) = -\frac{4}{c^2}\,\omega^{(a)} n_{(a)} \cdot 2\mathcal{A} + \mathcal{O}\left(A_{(a)}^{\;(i_1)(i_2)}\right)$$

DELI: Measurement mode 2



$$\Delta\tau_{Sp}(\mathcal{S}_1) = n_{(a)}\left[\frac{8}{3c}\varepsilon^{(0ajk)}R_{(0)\{(i_1)(j)\}(k)}(p^{(r)}) + \frac{4}{c^4}\left(\omega^{(l)}a_{(l)}\delta^{(a)}_{(i_1)} - 3\omega^{(a)}a_{(i_1)}\right)\right] m^{(i_1)}\cdot 2\mathcal{M}$$

Sagnac effect in a rotating reference frame (flat spacetime) and in Gödel's spacetime

	DELI operation mode 1	DELI operation mode 2
Flat spacetime		
$\Delta\tau_{Sp}$	$\frac{4}{c^2}\mathcal{N}\,\Omega_R\, n^{(3)}\cdot 2\mathcal{A}$	$\frac{12}{c^4}\mathcal{N}^3\, r_0'\,\Omega_R^3\, n^{(3)}\, m^{(1)}\cdot 2\mathcal{M}$
Gödel's spacetime		
$\Delta\tau_{Sp}$	$-\frac{4}{c^2}\Omega_G\, n^{(3)}\cdot 2\mathcal{A}$	0

with constants: $\Omega_G = \frac{c}{\sqrt{2}a}$ $\qquad$ $\mathcal{N} = \frac{1}{\sqrt{1-\left(\frac{r_0'\Omega_R}{c}\right)^2}}$

The team

(Institute of Quantum Physics, Ulm University)

Michael Buser

Cornelia Feiler

Wolfgang P. Schleich

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F. Grave, T. Müller, H. Ruder, G. Wunner.
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The team
(Institute of Quantum Physics, Ulm University)

Michael Buser

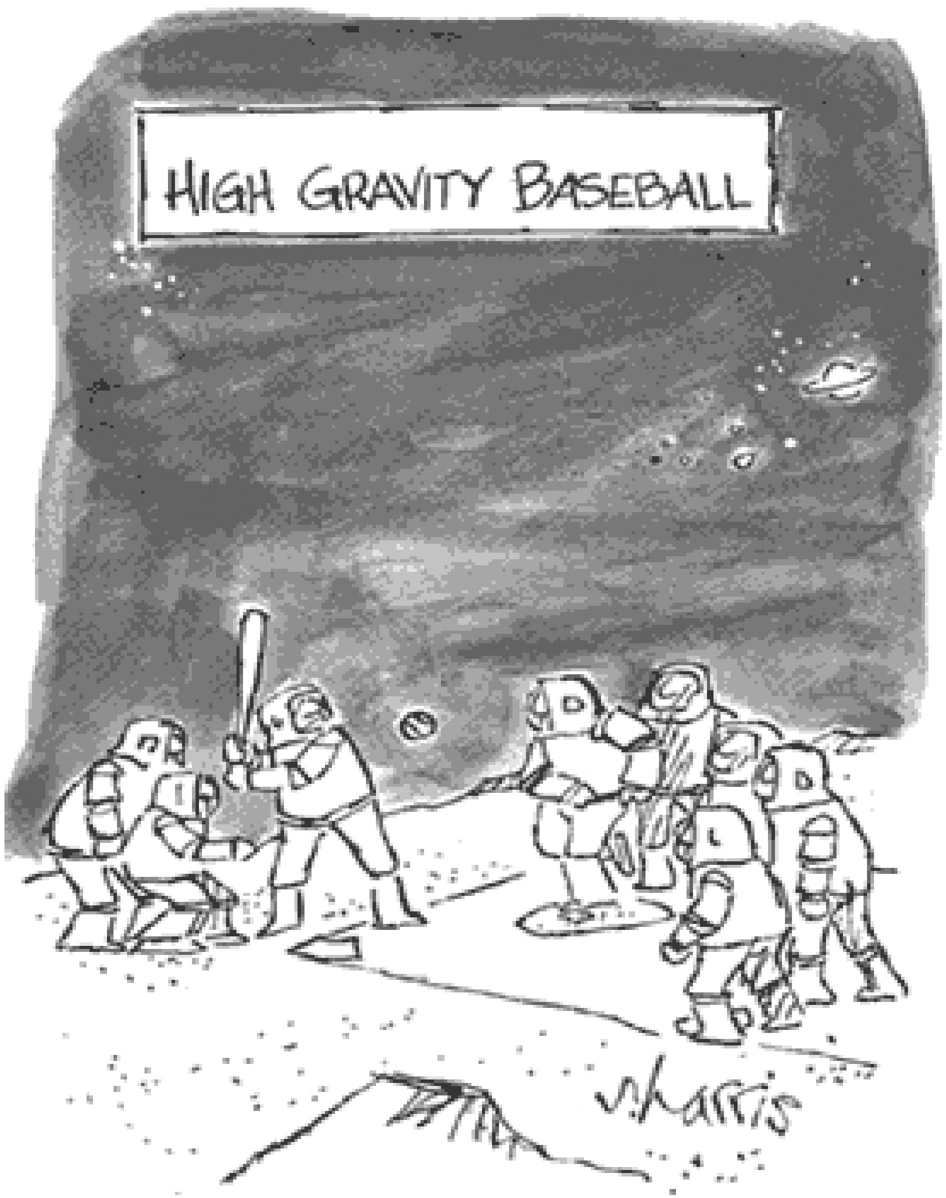
Cornelia Feiler

Wolfgang P. Schleich

E. Kajari, M. Buser, C. Feiler and W. P. Schleich,
"Rotation in relativity and the propagation of light" in the Proceedings of the International School of Physics "Enrico Fermi" Course "Atom Optics and Space Physics" (2009)

Thank you very much

for your attention!



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