Experimental Detection of the Gravitomagnetic Field of Moving Masses

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We are going to discuss:

1) Two types of gravitomagnetic field
2) Parameterization of gravitomagnetic effects
3) Gravitational time delay by a moving mass
4) Lorentz-covariance of the gravitational time delay
5) Speed-of-gravity experiment (2002)
6) Interpretations of the speed-of-gravity experiment
7) VLBA experiment with Saturn (2009)
8) Gravitomagnetism in Cassini experiment

Gravitomagnetism

GRAVITOMAGNETIC FIELD arises from moving masses just as a magnetic field arises from moving electric charges.

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

The metric tensor

$$\Phi \equiv \frac{c^2}{2} h_{00}$$

The gravitoelectric potential
The leading term is U=GM/r.

$$A_i \equiv -\frac{c^2}{4} h_{0i}$$

The gravitomagnetic potential
The leading term is (v/c)U.

Two types of gravitomagnetic field

Intrinsic (Lense-Thirring):

rotating currents of matter induced by angular momentum of the massive body

Extrinsic (Lorentz-Einstein):

translational currents of matter induced by motion of the massive body in space with respect to observer or another body

Lense-Thirring force on a test particle

- LAGEOS, LARES - spin-orbit LT force in motion of point-like test particles

$$\frac{d\mathbf{v}}{dt} = -\frac{GM_{\oplus}}{r^3}\mathbf{r} + \mathbf{F}_{\text{gm}}^{\text{intrinsic}} + \mathbf{F}_{\text{noise}} \qquad \text{where} \qquad \mathbf{F}_{\text{gm}}^{\text{intrinsic}} = \underbrace{\frac{4}{c}\mathbf{v}\times\left(\nabla\times\mathbf{A}_{\oplus}\right)}_{\substack{\text{post-Newtonian force} \\ \text{of the order of } 1/c^2}}$$

- GP-B - spin-spin LT interaction in motion of spinning test particles (gyroscopes)

$$\frac{d\mathbf{s}}{dt} = \mathbf{T}_{\text{gm}}^{\text{intrinsic}} + \mathbf{T}_{\text{noise}} \qquad \text{where} \qquad \mathbf{T}_{\text{gm}}^{\text{intrinsic}} = \underbrace{\frac{4}{c}\mathbf{s}\times\left(\nabla\times\mathbf{A}_{\oplus}\right)}_{\substack{\text{post-Newtonian torque} \\ \text{of the order of } 1/c^2}}$$

Extrinsic gravitomagnetic force on a test particle

$$\frac{d\mathbf{v}}{dt} = \left(1 + \frac{\mathbf{v}^2}{c^2}\right)\nabla\Phi - \frac{4}{c^2}\mathbf{v}\left(\mathbf{v}\cdot\nabla\Phi\right) + \mathbf{F}_{\text{gm}}^{\text{extrinsic}} + \mathbf{F}_{\text{noise}}$$

$$\mathbf{F}_{\text{gm}}^{\text{extrinsic}} = \frac{4}{c}\mathbf{v}\times\left(\nabla\times\mathbf{A}\right) + \underbrace{\frac{4}{c}\frac{\partial\mathbf{A}}{\partial t} - \left[\left(3 - \frac{\mathbf{v}^2}{c^2}\right)\left(\frac{1}{c}\frac{\partial\Phi}{\partial t}\right) - 4\left(\frac{\mathbf{v}}{c}\cdot\nabla\right)\left(\frac{\mathbf{v}}{c}\cdot\mathbf{A}\right)\right]\frac{\mathbf{v}}{c} - \frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\nabla\chi}_{\text{these terms vanish in the field of a rotating mass being at rest}}$$

Massive body must move wrt observer to generate the extrinsic GM. How to measure it?

USE PHOTONS ! For photons $\mathbf{v} = c\mathbf{k}$ that amplifies the PN terms depending on v/c = O(1)

$$c\frac{d\mathbf{k}}{dt} = \underbrace{2\nabla\Phi - 4\mathbf{k}\left(\mathbf{k}\cdot\nabla\Phi\right)}_{\text{"Newtonian" force}} + \mathbf{F}_{\text{gm}}^{\text{extrinsic}} + \mathbf{F}_{\text{noise}}$$

$$\mathbf{F}_{\text{gm}}^{\text{extrinsic}} = \underbrace{4\mathbf{k}\times\left(\nabla\times\mathbf{A}\right) - \mathbf{k}\left[\frac{2}{c}\frac{\partial\Phi}{\partial t} - 4\left(\mathbf{k}\cdot\nabla\right)\left(\mathbf{k}\cdot\mathbf{A}\right)\right]}_{\text{post-Newtonian force of the order of V}/c} \underbrace{+\frac{4}{c}\frac{\partial\mathbf{A}}{\partial t} - \frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\nabla\chi}_{\text{post-Newtonian force of the order of V}^2/c^2}$$

Post-Newtonian Parameterization of Gravitomagnetism

$$\boldsymbol{j} \to \epsilon \boldsymbol{j} \,, \qquad \boldsymbol{v} \to \epsilon \boldsymbol{v} \,, \qquad \frac{\partial}{\partial t} \to \epsilon \frac{\partial}{\partial t}$$

Gravity Fields

$$\boldsymbol{E} = -\boldsymbol{\nabla}\Phi - \frac{\epsilon}{c}\frac{\partial \boldsymbol{A}}{\partial t} \,,$$
$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \,,$$

Gauge condition

$$\frac{\epsilon}{c}\frac{\partial \Phi}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0 \,,$$

Einstein's Field Equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi G \rho \,,$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \,,$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\epsilon}{c}\frac{\partial \boldsymbol{B}}{\partial t} \,,$$
$$\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\epsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi G \epsilon}{c}\boldsymbol{j} \,,$$

Post-Newtonian parameter ε labels time-dependent gravitational effects and characterizes the speed of the respond of gravitational field to the positional changes of the massive body. It can be conveniently replaced with a "speed of gravity" parameter $c_g = c / \varepsilon$

Hence, $\varepsilon = \dfrac{c}{c_g}$

Lorentz Transform and Gravitomagnetism

$$h_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha}(\boldsymbol{\beta}_{\varepsilon})\Lambda^{\nu}{}_{\beta}(\boldsymbol{\beta}_{\varepsilon})h'_{\mu\nu} \qquad \boldsymbol{\beta}_{\varepsilon} = \varepsilon\frac{\mathbf{v}}{c}$$

$$\Phi = \gamma_{\epsilon}^{2}\left[\left(1+\beta_{\epsilon}^{2}\right)\Phi' + 4\left(\boldsymbol{\beta}_{\epsilon}\cdot\boldsymbol{A}'\right)\right] \ ,$$

$$\boldsymbol{A} = \gamma_{\epsilon}\boldsymbol{A}' + \gamma_{\epsilon}^{2}\left[\Phi' + \frac{2\gamma_{\epsilon}+1}{\gamma_{\epsilon}+1}\left(\boldsymbol{\beta}_{\epsilon}\cdot\boldsymbol{A}'\right)\right]\boldsymbol{\beta}_{\epsilon} \ ,$$

When massive body is at rest the gravitomagnetic potentials and fields are given by:

$\Phi' = -GM/c^2r'$, $\boldsymbol{A}' = 0$, and the fields: $\boldsymbol{E}' = -\boldsymbol{\nabla}\Phi' \neq 0$, $\boldsymbol{B}' = \boldsymbol{\nabla}\times\boldsymbol{A}' = 0$

When the massive body is moving the gravitomagnetic potentials and fields are:

$$\Phi = -\frac{GM}{c^2 r} + O\left(\epsilon^2\right) \ , \qquad \boldsymbol{A} = \frac{\epsilon}{c}\Phi\boldsymbol{v} + O\left(\epsilon^2\right) \ ,$$

$$\boldsymbol{B} = \frac{\epsilon}{c}\left(\boldsymbol{v}\times\boldsymbol{E}\right) \qquad \text{Moreover,} \quad \frac{\partial\Phi}{\partial t} \neq 0 \ \text{ and } \ \frac{\partial\mathbf{A}}{\partial t} \neq 0$$

Gravitational Time Delay

$$\frac{d^2x^i}{dt^2} = c^2 k^\mu k^\nu \left(k^i \Gamma^0_{\mu\nu} - \Gamma^i_{\mu\nu}\right)$$

$$\begin{aligned}
\Gamma^0_{00} &= -\frac{\epsilon}{c}\frac{\partial \Phi}{\partial t} , \\
\Gamma^0_{0i} &= -\frac{\partial \Phi}{\partial x^i} , \\
\Gamma^0_{ij} &= +2\left(\frac{\partial A^j}{\partial x^i} + \frac{\partial A^i}{\partial x^j}\right) + \frac{\epsilon}{c}\frac{\partial \Phi}{\partial t}\delta_{ij} , \\
\Gamma^i_{00} &= -\frac{\partial \Phi}{\partial x^i} - 4\frac{\epsilon}{c}\frac{\partial A^i}{\partial t} , \\
\Gamma^i_{0j} &= -2\left(\frac{\partial A^i}{\partial x^j} - \frac{\partial A^j}{\partial x^i}\right) + \frac{\epsilon}{c}\frac{\partial \Phi}{\partial t}\delta_{ij} , \\
\Gamma^i_{jp} &= -\delta_{jp}\frac{\partial \Phi}{\partial x^i} + \delta_{ip}\frac{\partial \Phi}{\partial x^j} + \delta_{ij}\frac{\partial \Phi}{\partial x^p} ,
\end{aligned}$$

Parameterized Time Delay Equation

$$t_1 - t_0 = \frac{1}{c} | \mathbf{x}_1 - \mathbf{x}_0 | + \Delta(t_1, t_0)$$

$$\mathbf{x}_N(t) = \mathbf{x}_0 + c\mathbf{k}(t - t_0)$$

$$\Delta(t_1, t_0) = \frac{1}{2}\int_{t_0}^{t_1} dt \left\{ k^\alpha k^\beta h_{\alpha\beta}(t, \mathbf{x}_N(t)) + (\varepsilon - 1)\int_{t_0}^{t} d\sigma k^\alpha k^\beta \left[\frac{\partial h_{\alpha\beta}(\sigma, \mathbf{x})}{\partial \sigma} \right]_{\mathbf{x}=\mathbf{x}_N(\sigma)} \right\}$$

References for further particular details and experimental applications:

Kopeikin S. (2004) Class. Quant. Grav., 21, 3251

Kopeikin S. (2006) Int. J. Mod. Phys. D, 15, 305

Kopeikin S. & Fomalont E. (2006) Found. Phys., No. 1, pp. 1 - 42

Kopeikin S & Ni W.-T. (2006) In: Proc. 359th WE-Heraeus Seminar

Kopeikin & Makarov (2007) Phys. Rev. D, 75, 062002

Fomalont et al (2009) in: Proc. IAU Symp 261

Gravitational Time Delay by a moving body

$$h_{00} = \frac{2GM}{|\mathbf{x} - \mathbf{z}(t)|} \qquad h_{ij} = \frac{2GM\,\delta_{ij}}{|\mathbf{x} - \mathbf{z}(t)|} \qquad h_{0i} = \frac{4GM}{|\mathbf{x} - \mathbf{z}(t)|}\left(\frac{\mathbf{v}}{c_g}\right)$$

photon: $\mathbf{x} \mapsto \mathbf{x}_N(t) = \mathbf{x}_0 + c\mathbf{k}(t - t_0)$ massive body: $\mathbf{z}(t) = \mathbf{z}_0 + \mathbf{v}(t - t_0)$

$$\Delta(t_1, t_0) = 2\frac{GM}{c^3}\left(1 - \frac{1}{c_g}\mathbf{k}\cdot\mathbf{v}\right)\ln\left[\frac{|\mathbf{x}_1 - \mathbf{z}(s_1)| - \mathbf{k}\cdot\left(\mathbf{x}_1 - \mathbf{z}(s_1)\right)}{|\mathbf{x}_0 - \mathbf{z}(s_0)| - \mathbf{k}\cdot\left(\mathbf{x}_0 - \mathbf{z}(s_0)\right)}\right]$$

$$\mathbf{z}(s_1) = \mathbf{z}(t_1) - \frac{\mathbf{v}}{c_g}|\mathbf{x}_1 - \mathbf{z}(t_1)| + O\left(\frac{v^2}{c_g^2}\right) \qquad \mathbf{z}(s_0) = \mathbf{z}(t_0) - \frac{\mathbf{v}}{c_g}|\mathbf{x}_0 - \mathbf{z}(t_0)| + O\left(\frac{v^2}{c_g^2}\right)$$

Look like a retarded time

$$s_1 = t_1 - \frac{1}{c_g}|\mathbf{x}_1 - \mathbf{z}(t_1)| \qquad s_0 = t_0 - \frac{1}{c_g}|\mathbf{x}_0 - \mathbf{z}(t_0)|$$

Lienard-Wichert gravitational potentials

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2 \right) h^{\alpha\beta} = -\frac{16\pi G}{c^4}\left(T^{\alpha\beta} - \frac{1}{2}\eta^{\alpha\beta} T \right) \quad \text{where } T^{\alpha\beta} = \frac{Mc^2}{\gamma} u^\alpha(t) u^\beta(t)\ \delta\left[\mathbf{x} - \mathbf{z}(t)\right]$$

Lienard-Wiechert retarded (gravitational) potentials

$$h_{\alpha\beta} = \frac{2GM}{c^2}\frac{2u^\alpha u^\beta + \eta^{\alpha\beta}}{u_\mu r^\mu}$$

where $r^\mu = (r, \mathbf{r})$ is a null vector in Minkowski space-time connecting the moving massive body to the field point,

$r = |\mathbf{r}|$; $\mathbf{r} = \mathbf{x} - \mathbf{z}(s)$; $u^\alpha = \gamma\left(1, \frac{\mathbf{v}}{c}\right)$ all functions depend on the retarded time s

$s = t - \frac{1}{c}|\mathbf{x} - \mathbf{z}(s)|$ due to the Lorentz-invariant nature of gravity field (a finite speed of gravity)

Lorentz-invariant gravitational time delay

Substitute the Lienard-Wiechert retarded (gravitational) potentials to the light geodesic equation and integrate along the light ray. It yields the Lorentz-covariant gravitational time delay equation (Kopeikin et al 1999, 2002)

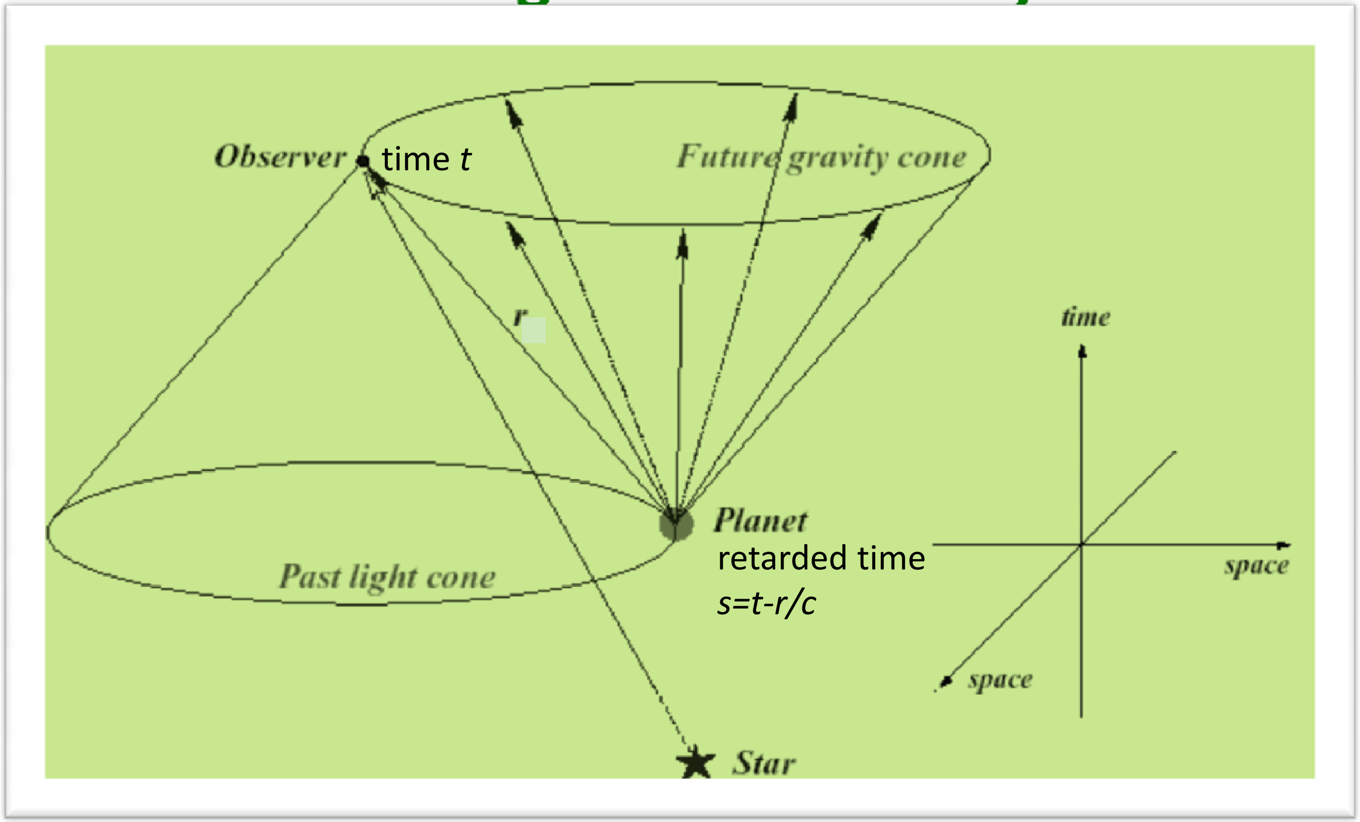
$$\Delta(t_1,t_0) = 2\frac{GM}{c^3}\frac{1-\frac{1}{c}\mathbf{k}\cdot\mathbf{v}}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}\ln\left[\frac{|\mathbf{x}_1-\mathbf{z}(s_1)|-\mathbf{k}\cdot\left(\mathbf{x}_1-\mathbf{z}(s_1)\right)}{|\mathbf{x}_0-\mathbf{z}(s_0)|-\mathbf{k}\cdot\left(\mathbf{x}_0-\mathbf{z}(s_0)\right)}\right] ,$$

where $\mathbf{k}$ is the light vector the retarded time arguments

$$s_1 = t_1 - \frac{1}{c}|\mathbf{x}_1-\mathbf{z}(s_1)| \qquad\qquad s_0 = t_0 - \frac{1}{c}|\mathbf{x}_0-\mathbf{z}(s_0)|$$

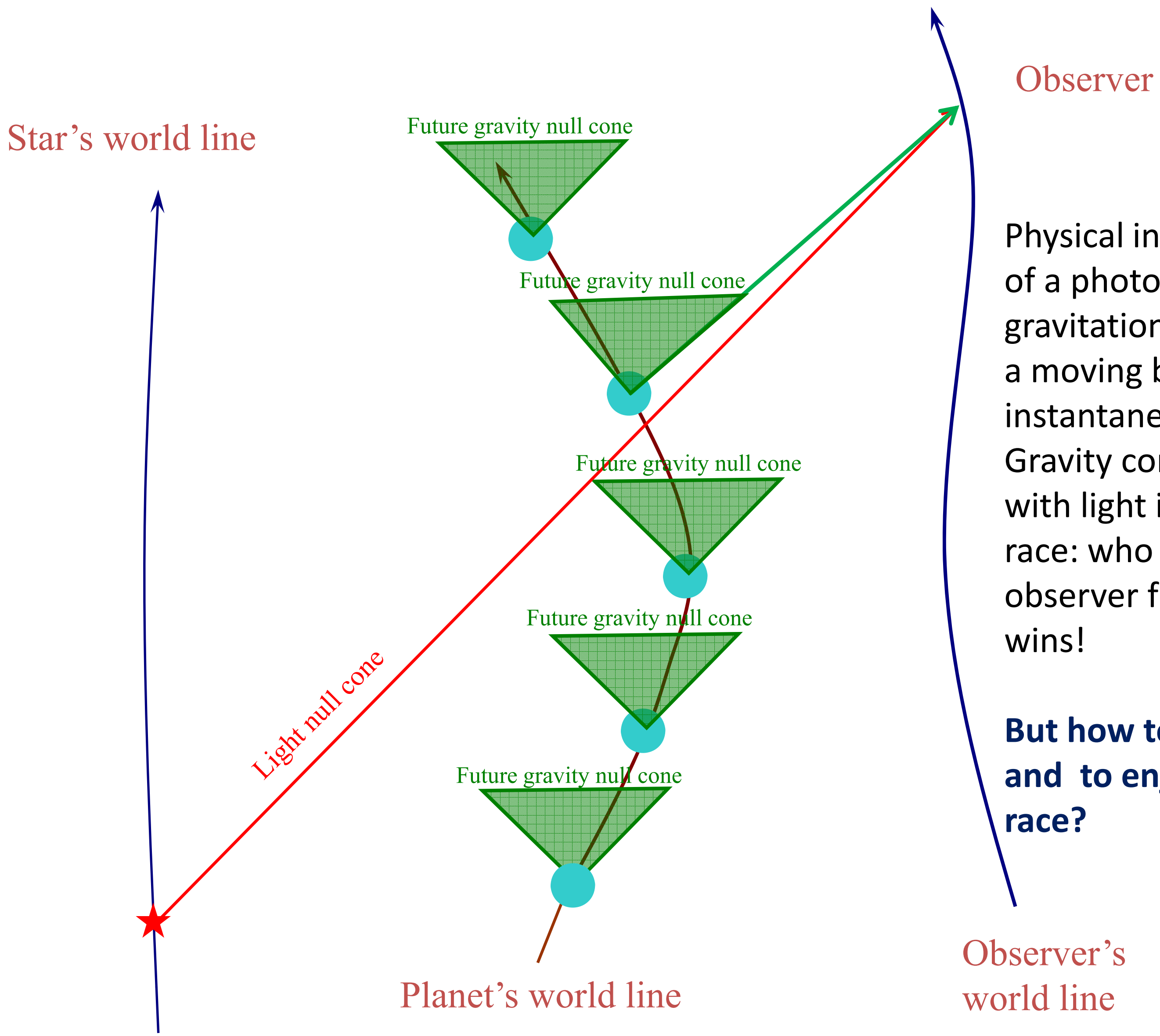
tracks down the null-cone (gravitomagnetic) effects of gravity.
Compare with the previous solution of the linearized order of v/c beyond the static time delay approximation. Make physical interpretation

Bi-characteristic interaction of gravity and light in general relativity



Any type of gravitational field obeys the principle of causality. Even the slowly evolving "Coulomb component" of the gravitational field of a moving body can not transfer information about position of the body with the speed faster than the speed of light.

Light and gravity null cones



Physical interaction of a photon with gravitational field of a moving body is not instantaneous. Gravity competes with light in the race: who will reach observer first. None wins!

But how to watch and to enjoy the race?

The speed-of-gravity experiment (2002)

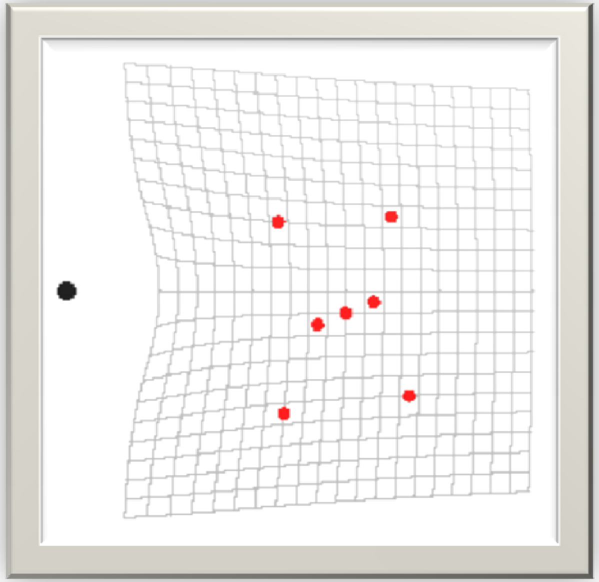
Edward B. Fomalont
(observation + data processing)

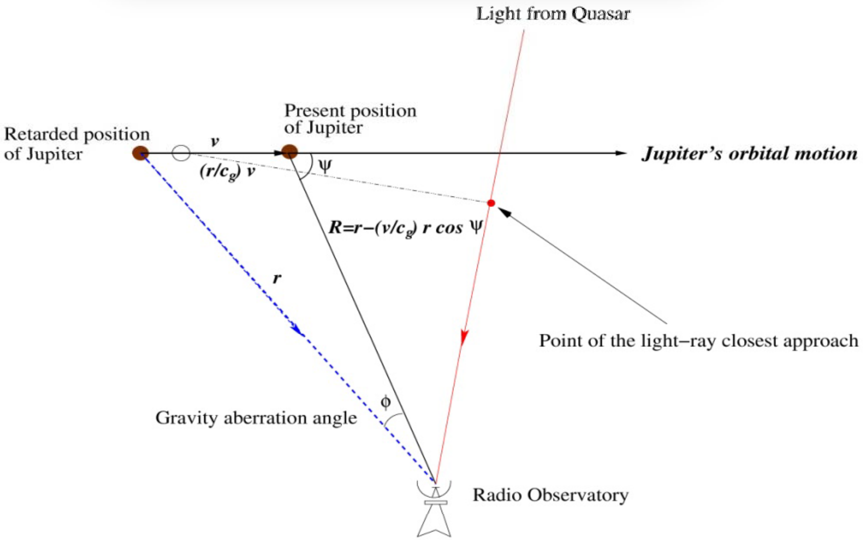
Sergei M. Kopeikin
(theory + interpretation)

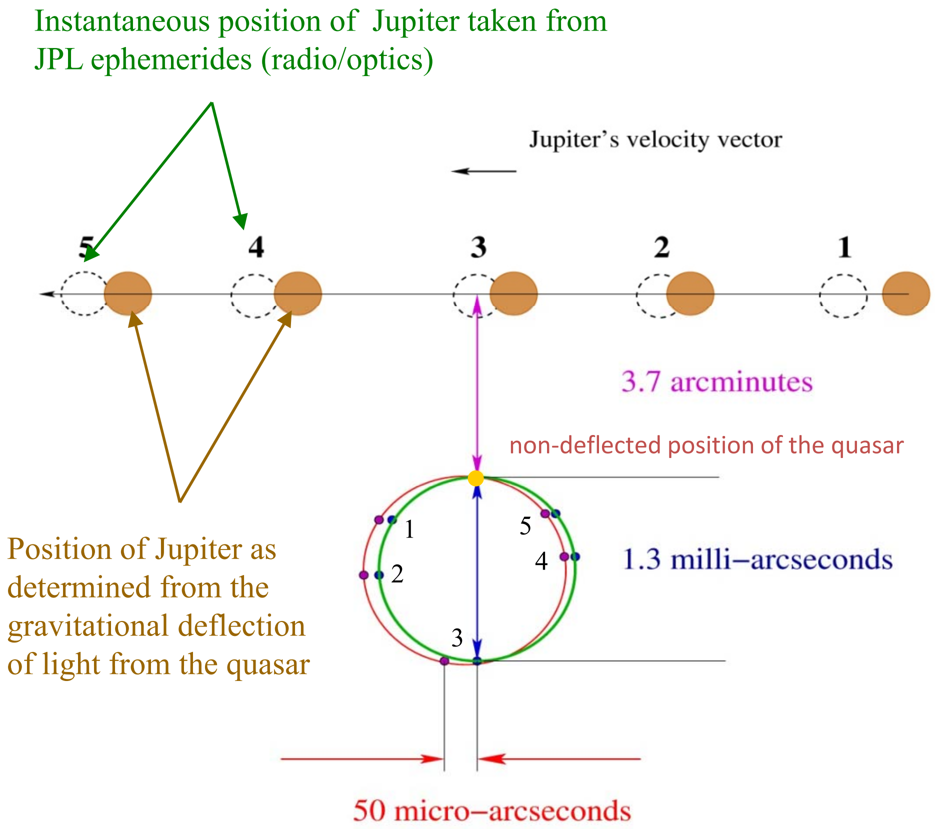
(with support of NRAO, MPIfR
and the NRAO correlator staff)

The speed-of-gravity VLBI experiment (September 2002)

Fomalont & Kopeikin, ApJ., 598, 704 (2003)   **Jupiter- J0842+1835**



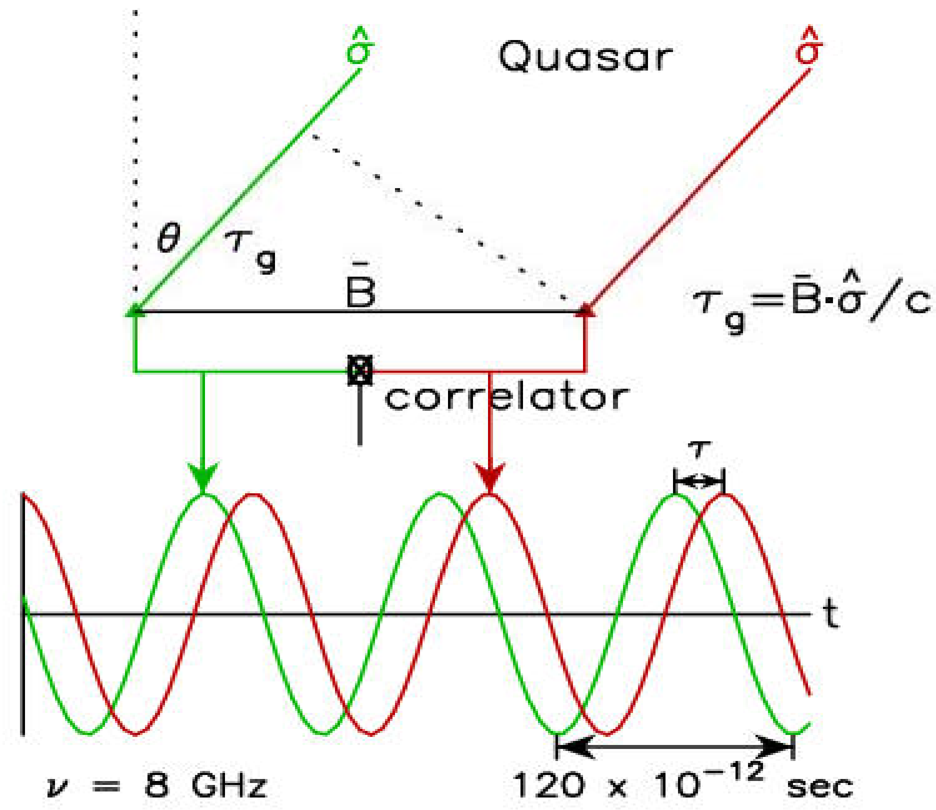




Measured with 20% of accuracy, thus, proving that the fundamental speed of gravity equals the speed of light (the null cones for gravity and light are identical)

10 microarcseconds = the width of a typical strand of a human hair from a distance of 650 *miles*!!!

BASIC INTERFEROMETRY



τ_g (B=5000 km) $= 2 \times 10^{-2}$ sec

$\delta\tau$ $(0.02/\nu) = 2 \times 10^{-12}$ sec

$\delta\tau / \tau_g = 10^{-10}$ rad = 20 μarcsec (in one minute)

Limitations to Positional Accuracy

- Location of Radio Telescope
  Position on earth (**1 cm**)
  Earth Rotation and orientation (**5 cm**)
- Time synchronization (**50 psec**)
- Array stability (**5 cm**)
- Propagation in troposphere and ionosphere
  Very variable in time and space (**5 cm in 10 min**)

CONVERSION FACTORS for astrometry:

1 cm = 30 psec = 300 microarcsec

0.03cm = 1 psec = 10 microarcsec

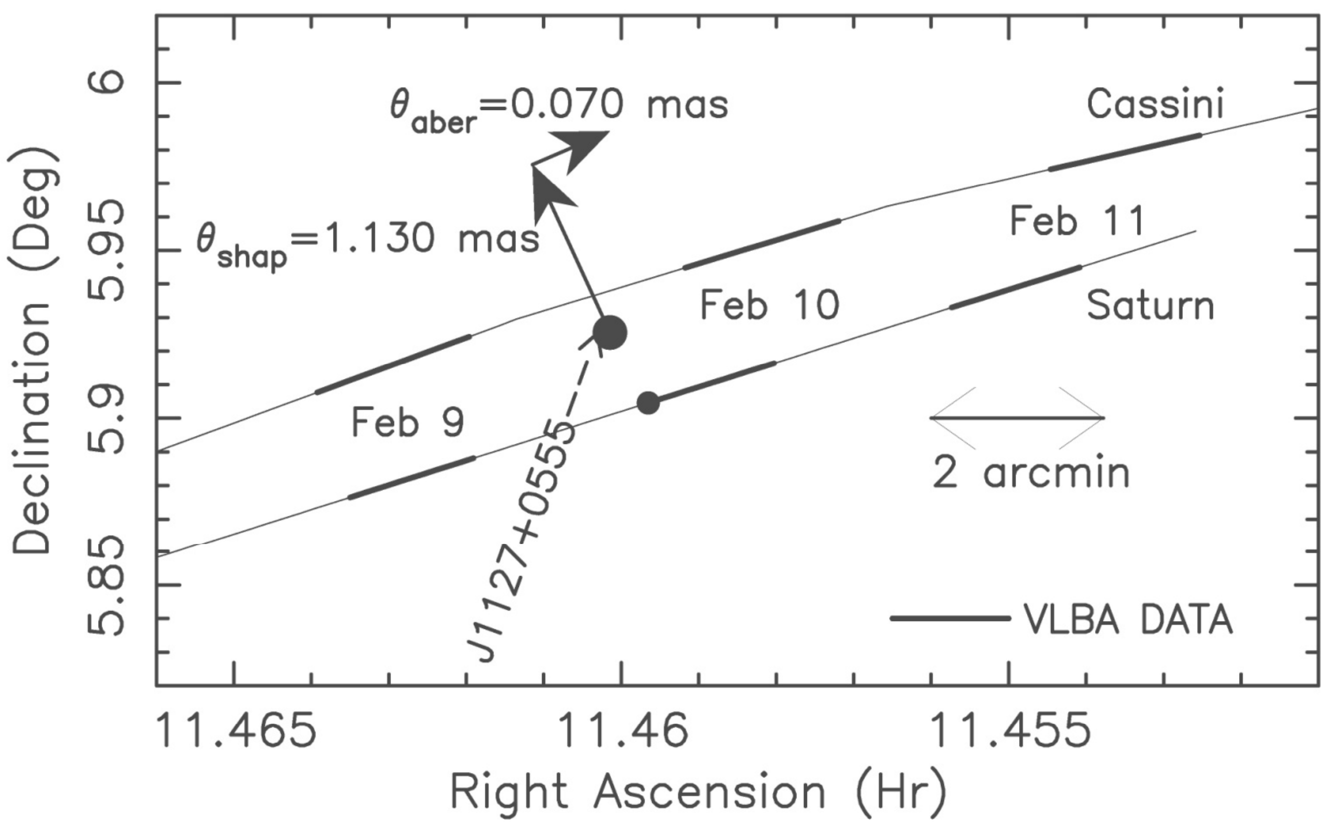
Phase-referencing VLBI technique can achieve 10 microarcsec!

Interpreting the Jovian experiment

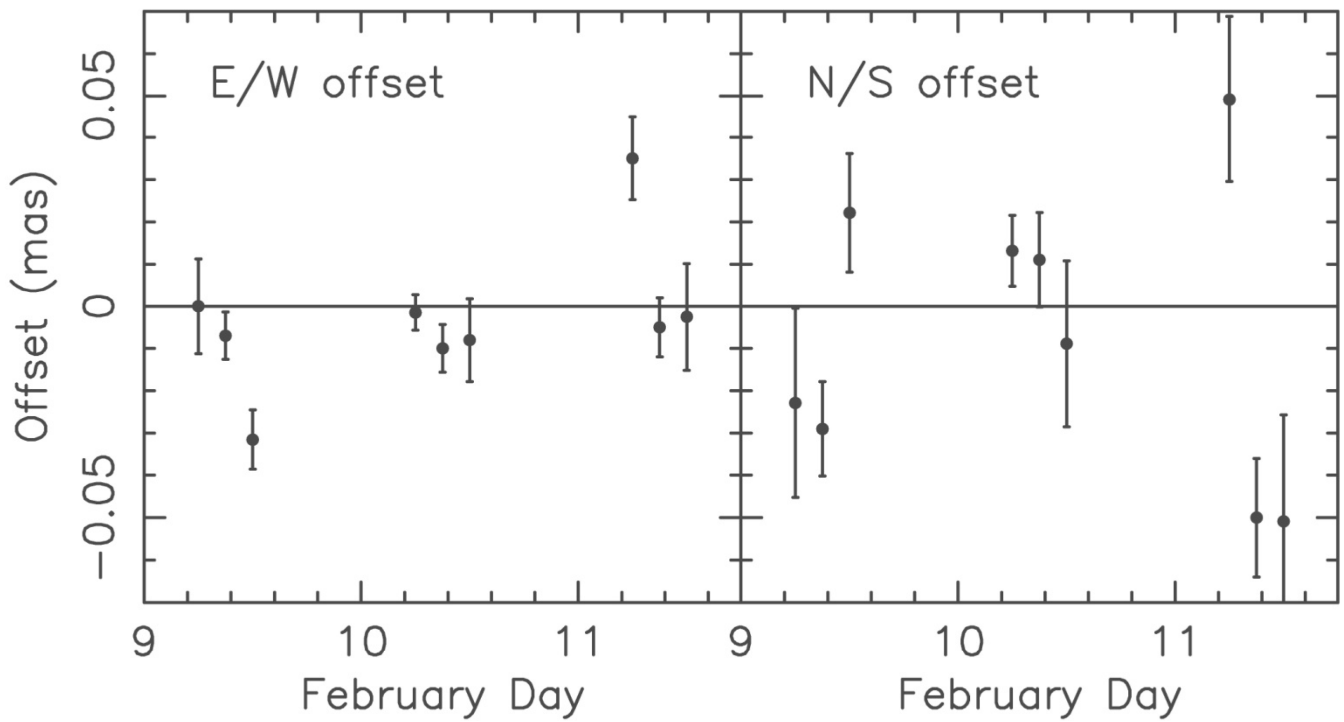
Kopeikin & Fomalont - null characteristic of Einstein's field equations,
speed of gravity = speed of light [$\varepsilon = 1$]
gravitomagnetic field of moving Jupiter (time derivatives of the metric tensor)
aberration of the gravity force [it's $(v/c)^2$ for slowly-moving particles but becomes v/c for photons!]
causal behavior of gravity

1. **Will** – aberration of light (radio waves) from the quasar
2. **Asada, Carlip** – speed of light (radio waves) from the quasar
3. **Nordtvedt** – retardation of radio waves from the quasar as they pass through the Jovian magnetosphere
4. **Pascual-Sanchez** – the Römer delay of light that has been already known since 1676
5. **Samuel** – retardation of radio waves emitted by Jupiter itself
6. **Van Flandern** – the quantity measured was already known to propagate at the speed of light
7. **Van Nieuwenhuizen** – "compete nonsense"

Light Deflection Experiment with Saturn (February 2009)

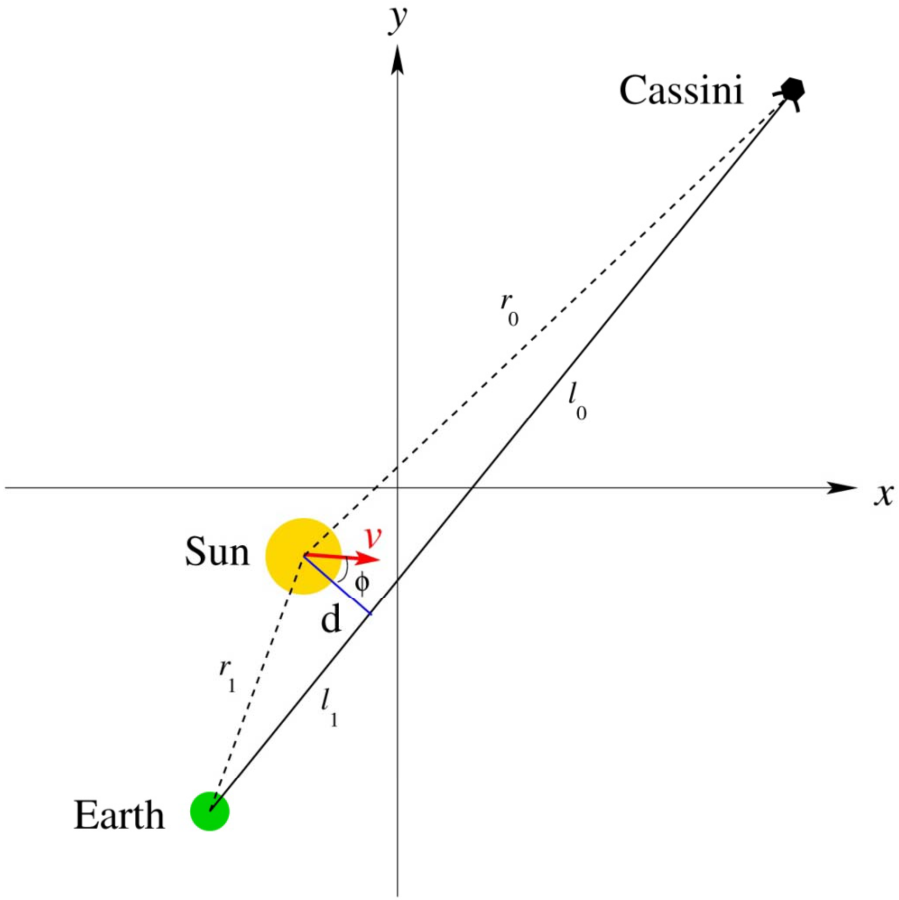


R.M.S. residuals of the light deflection experiment with Saturn (Fomalont, Kopeikin et al, Proc. IAU Symp. 261, 2009)



Gravitomagnetic Field in Cassini Experiment
(Kopeikin et al., Phys. Lett. A, 2007)

Gravitomagnetic Doppler shift due to the orbital motion of the Sun



$$z_{gr} = -\underbrace{\frac{l_0}{cr}(\boldsymbol{v}_1 \cdot \boldsymbol{\alpha}_B)}_{\text{observer shift } z_O} - \underbrace{\frac{l_1}{cr}(\boldsymbol{v}_0 \cdot \boldsymbol{\alpha}_B)}_{\text{satellite shift } z_S} + \underbrace{\frac{1}{c}(\boldsymbol{v}_\odot \cdot \boldsymbol{\alpha}_B)}_{\text{gravimagnetic shift } z_{GM}}$$

$$\boldsymbol{\alpha}_B = \alpha_\odot \frac{1+\gamma}{2}\frac{R_\odot}{d}\hat{\boldsymbol{d}}\,,$$

1.″7505

Bertotti-Iess-Tortora, Nature, 2004
$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$
However, the gravitomagnetic contribution was not analyzed

Gravitational time delay in the ODP code

The linearized w.r.t. v/c time delay equation can be re-formulated as follows (*Kopeikin* arXiv:0809.3433)

$$\Delta_{\text{Cassini-Earth}} = 2\frac{GM}{c^3}\left(1-\frac{1}{c}\mathbf{k}\cdot\mathbf{v}\right)\ln\left[\frac{R_1+R_2+R_{12}}{R_1+R_2-R_{12}}\right]$$

$$\mathbf{R}_1 = \mathbf{x}_1 - \mathbf{z}(t_1) \qquad \mathbf{R}_2 = \mathbf{x}_2 - \mathbf{z}(t_2) \qquad R_{12} = |\mathbf{R}_1 - \mathbf{R}_2|$$

$$\mathbf{z}(t_1) = \mathbf{z}_0 + \mathrm{v}(t_1 - t_0) \qquad \mathbf{z}(t_2) = \mathbf{z}_0 + \mathbf{v}(t_2 - t_0)$$

Notice that velocity $\mathbf{v}$ of the light-ray deflecting body enters the argument of the logarithm in the time delay.

Numerical Estimates for Cassini Doppler Shift

- The peak value of the Doppler shift is caused by orbital motion of Earth and reaches 6×10^{-10}.
- R.M.S. error of the measurements is $\pm1\times10^{-14}$
- Doppler shift due to the orbital motion of Sun is 2.9×10^{-13}
- The value of (γ-1) would be affected by the solar motion by the amount $\pm1.2\times10^{-4}$ if the gravitomagnetic deflection of light were not in accordance with GR

Conclusions

1. Cassini solar conjunction experiment has a potential to detect the gravitomagnetic field of the moving Sun directly!
2. It requires re-processing of the data
3. The announced value for $\gamma-1=(2.1\pm2.3)\times10^{-5}$ is based on *implicit* assumption that the gravitomagnetic deflection of light agrees with GR, but this assumption has not yet been tested.

Gracias !

Does LLR measure a gravitomagnetic field?

Murphy, Nordtvedt, Turyshev, PRL (2007):

YES – the EIH equations for the Moon show explicitly the "gravitomagnetic" term. It leads to the coordinate oscillations of the lunar orbit amounting to a few meters in analytic solution of these equations. The "gravitomagnetic" oscillations are not visible in LLR data residuals. However, if the "gravitomagnetic" term is removed from the EIH equations, the numerical solution of the equations can not fit to the data. This discrepancy means that LLR measures the "gravitomagnetic" term.

Hans Christian Anderson **The Emperor's New Clothes**

Kopeikin, PRL (2007)

NO - the LLR technique involves processing data with two sets of mathematical equations, one related to the motion of Moon around Earth, and the other related to the propagation of the laser beam from Earth to Moon. These equations can be written in different ways based on "gauge freedom", the idea that arbitrary coordinates can be used to describe gravitational physics. Scrutiny analysis of the gauge freedom of the LLR technique shows that the computer manipulation of the mathematical equations is causing NASA scientists to derive results that are not apparent in the data itself.