Local Physics and a Global Geometry

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Content of the lecture:

1. The ultimate goal of RCMA
2. Two spacetimes
3. Two physical principles
4. What time is physical?
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6. Newtonian limit in cosmology
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8. Light time non-uniformity
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10. The Hubble expansion in the solar system?
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The ultimate goal of celestial mechanics and astrometry (RCMA) is to study the fundamental structure of spacetime geometry underlying the world.

The truth of the geometry of space and time depends on the truth of axioms and, therefore, the question is whether the axioms are true.

There are two methods of testing the axioms of spacetime geometry:

- *Extrinsic* – look at the world “from the external” dimensions (high-energy physics, string theory, etc.)

- *Intrinsic* – measure the internal relationships between geometrical entities of the spacetime manifold

- RCMA in the solar system was built under an assumption that the background geometry is a static Minkowski spacetime

$$d\sigma^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

TCB (Barycentric Coordinate Time)

- On the other hand, the background geometry in cosmology is the RW spacetime ("expanding space")

$$ds^2 = -c^2dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

Physical time of the Hubble observers measured by clocks that are synchronized by means of Einstein's procedure of the exchange by light signals

$$a(t) = 1 + \mathcal{H}t + \tfrac{1}{2}\mathcal{H}'t^2 + \cdots$$

Hubble's parameter $\mathcal{H} = a'/a$

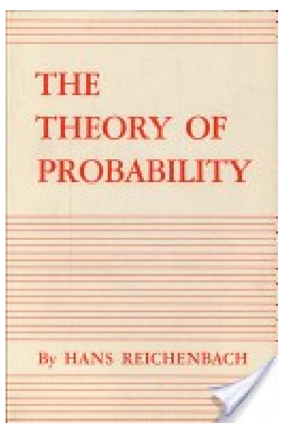
- ✓ Do the two geometries match?
- ✓ At what level?
- ✓ What principle(s) should we use to extrapolate RCMA from the Minkowski world to the expanding universe?
- ✓ Will the Newtonian equations be the same?
- ✓ Should we expect new effects?
- ✓ If, yes, are they observable?
- ✓ Can we find “non-expanding” standards of length in the “expanding space”?

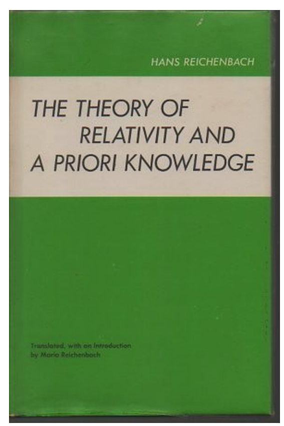
Causality – the driving principle

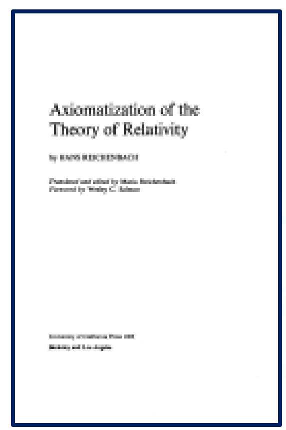
We arbitrarily can choose the geometry, or we arbitrarily can choose the causality; but we cannot choose both.

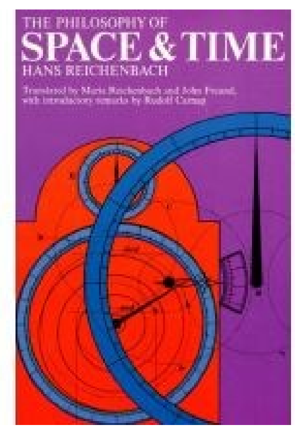
Hans Reichenbach

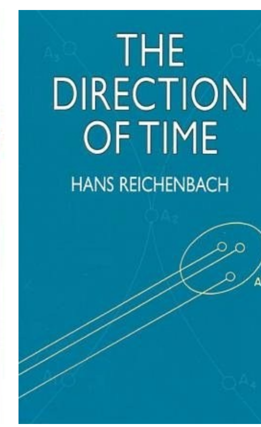
The Philosophy of Space and Time (1928)











Principle of equivalence versus the causality principle

For the RW cosmological spacetime two physical requirements

1) *Local causality is defined by the principle of equivalence,*

$$g_{\alpha\beta}(0) = \eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1); \qquad g_{\alpha\beta,\gamma}(0) = 0\,;$$

2) *Local causality is predetermined by the cosmological RW metric,*

are incompatible in a linear order with respect to the Hubble constant, $\mathcal{H} = a'/a$ **.**

Local diffeomorphism maps the RW metric to the Minkowski metric but it changes the time coordinate. The question is: what time is physical? The answer leads to different predictions.

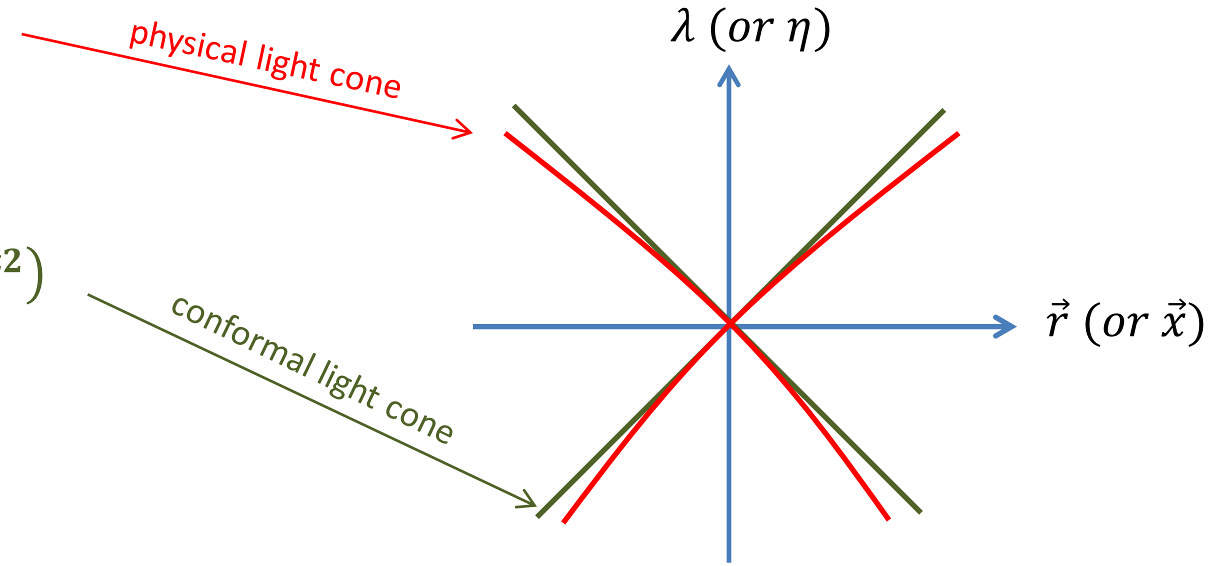
What time is physical?

Lemma: **two conformally-equivalent metrics have the same causal structure (identical light cones)**

$$\boldsymbol{ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2}$$

$$d\eta = \frac{dt}{a(t)}$$

$$\boldsymbol{ds^2 = a^2(t)\left(-c^2 d\eta^2 + d\vec{x}^2\right)}$$



$$\boldsymbol{ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2}$$

$$\lambda = t + \frac{\mathcal{H}\vec{x}^2}{2c^2} + O(\mathcal{H}^2)\,; \qquad \vec{r} = a(t)\vec{x} + O(\mathcal{H}^2)$$

$$\boldsymbol{ds^2 = -c^2 d\lambda^2 + d\vec{r}^2} + O(\mathcal{H}^2)$$

1) **Because the two "green" metrics are conformally equivalent, the coordinate time λ is a causal analogue of the conformal (*non-physical*) time η.**
2) **Physical time is the time of the Hubble observers, t. It is an analogue of TCB.**

Three Types of Physical Clocks in Cosmology

- **Atomic Clock (atomic time)**
  - ✓ is the electric charge conserved? **Yes**
  - ✓ is the Coulomb constant, $k = 1/4\pi\epsilon_0$, time-independent? **Yes**
  - ✓ have the electron orbitals a constant period (frequency)? **Yes**
- **Gravitational Clock (ephemeris time)**
  - ✓ are the mass, momentum, etc. conserved? **Yes**
  - ✓ is the universal gravitational constant, G , time-independent? **Yes**
  - ✓ have the planetary orbits constant periods? **Yes**
- **Light Clock (*light travels between "mirrors" - optical cavity*)**
  - ✓ does the optical length (range) remain the same? **Yes**
  - ✓ does the frequency of the traveling wave remain the same? **Yes**

Is there any "local" experiment at all that may trace the cosmological expansion? Many scientists say "no".

Newtonian limit of RCMA in cosmology

- **Solve Einstein equations on expanding cosmological manifold in conformal coordinates $(\eta, \vec{x})$. Messy business!**
- **Introduce physical coordinates $(t, \vec{r})$ where $\vec{r} = a(t)\vec{x}$.**
- **Transform equations of motion of massive bodies from the conformal to the physical coordinates.**
- **Take the Newtonian limit by making $c \to \infty$ in the equations.**

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^3}\vec{r}$$

Newton's law of gravity

This equation points out that the ephemeris time in the solar system is a physical realization of the cosmological time t.

Bohr's atom in cosmology

- **Solve Maxwell equations with sources on the expanding cosmological manifold in conformal coordinates $(\eta, \vec{x})$. Fairly strightforward!**
- **Right down the Lorentz force and the covariant equations of motion**
- **Introduce physical coordinates $(t, \vec{r})$ where $\vec{r} = a(t)\vec{x}$.**
- **Transform the equations of motion from the conformal to the physical coordinates.**
- **Take the Newtonian limit by making $c \to \infty$ in the equations.**

$$\frac{d^2\vec{r}}{dt^2} = \frac{k}{m_e}\frac{eQ}{r^3}\vec{r}$$

Coulomb's law of electrostatic

This equation points out that the atomic time is a physical realization of the cosmological time t.

Propagation of light in cosmology

- **Solve source-free Maxwell equations on expanding cosmological manifold in conformal coordinates $(\eta, \vec{x})$. Strightforward!**

$$\frac{d^2\vec{x}}{d\eta^2} = 0 \qquad \rightarrow \qquad \vec{x} = c\vec{k}\eta + \vec{x}_0$$

- **Introduce physical coordinates $(t, \vec{r})$ where $\vec{r} = a(t)\vec{x}$.**
- **Transform the light path from the conformal to the physical coordinates**

$$\frac{d^2\vec{r}}{dt^2} = \mathcal{H}\frac{d\vec{r}}{dt}$$

- **Expanding solution w.r.t. the Hubble parameter**

$$\vec{r} = c\vec{k}\left(t + \frac{1}{2}\mathcal{H}t^2\right) + \vec{r}_0$$

Light time non-uniformity

- Photons accelerate in physical coordinates $(t, \vec{r})$.
- A new time scale (re-parameterization of the light ray)

$$\lambda = a(\eta)\eta = t + \frac{1}{2}\mathcal{H}t^2$$

simplifies the light propagation equation to

$$\frac{d^2\vec{r}}{d\lambda^2} = 0$$

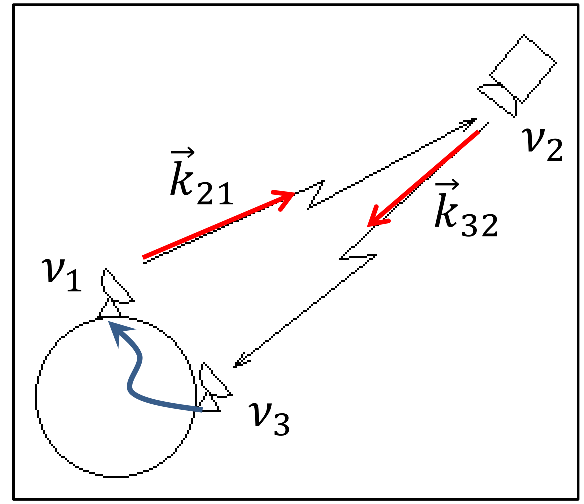
- Trajectory of light in an expanding universe in terms of λ time

$$\vec{r} = \vec{r}_0 + c\,\vec{k}\lambda \qquad\qquad \lambda_2 - \lambda_1 = \frac{1}{c}|\vec{r}_2 - \vec{r}_1|$$

- One can check that the quadratic-in-time t non-uniformity of the parameter λ affects ranging distances insignificantly (the effect is negligible). Therefore, it does not matter whether λ or t are used in the ranging equation to build celestial ephemerides.

Doppler tracking

$$\frac{\nu_2}{\nu_1} = \frac{1 + \mathcal{H}t_2 - \vec{k}_{21} \cdot \vec{\beta}_2}{1 + \mathcal{H}t_1 - \vec{k}_{21} \cdot \vec{\beta}_1} \qquad \frac{\nu_3}{\nu_2} = \frac{1 + \mathcal{H}t_3 - \vec{k}_{32} \cdot \vec{\beta}_3}{1 + \mathcal{H}t_2 - \vec{k}_{32} \cdot \vec{\beta}_2}$$



$$\frac{\nu_3}{\nu_1} = \frac{\nu_3}{\nu_2}\frac{\nu_2}{\nu_1} = 1 + 2\mathcal{H}(t_2 - t_1) - 2\vec{k}_{21} \cdot (\vec{\beta}_2 - \vec{\beta}_1)$$

Assuming that $\boldsymbol{t_3 - t_2 \simeq t_2 - t_1}$

Assuming that $\boldsymbol{\vec{k}_{32} \simeq -\vec{k}_{21}}$ and $\boldsymbol{\vec{\beta}_3 \simeq \vec{\beta}_1}$

Integrated Doppler tracking over a long time interval *t* *(a few years with current technology!)*

$$\frac{\nu_{2n+1}}{\nu_1} = \frac{\nu_{2n+1}}{\nu_{2n-1}}\frac{\nu_{2n-1}}{\nu_{2n-3}} \ldots \frac{\nu_3}{\nu_1} \Longrightarrow \frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{ref}}} = 1 + 2\mathcal{H}t - \frac{2}{c}\int_0^t a_r(t')dt'$$

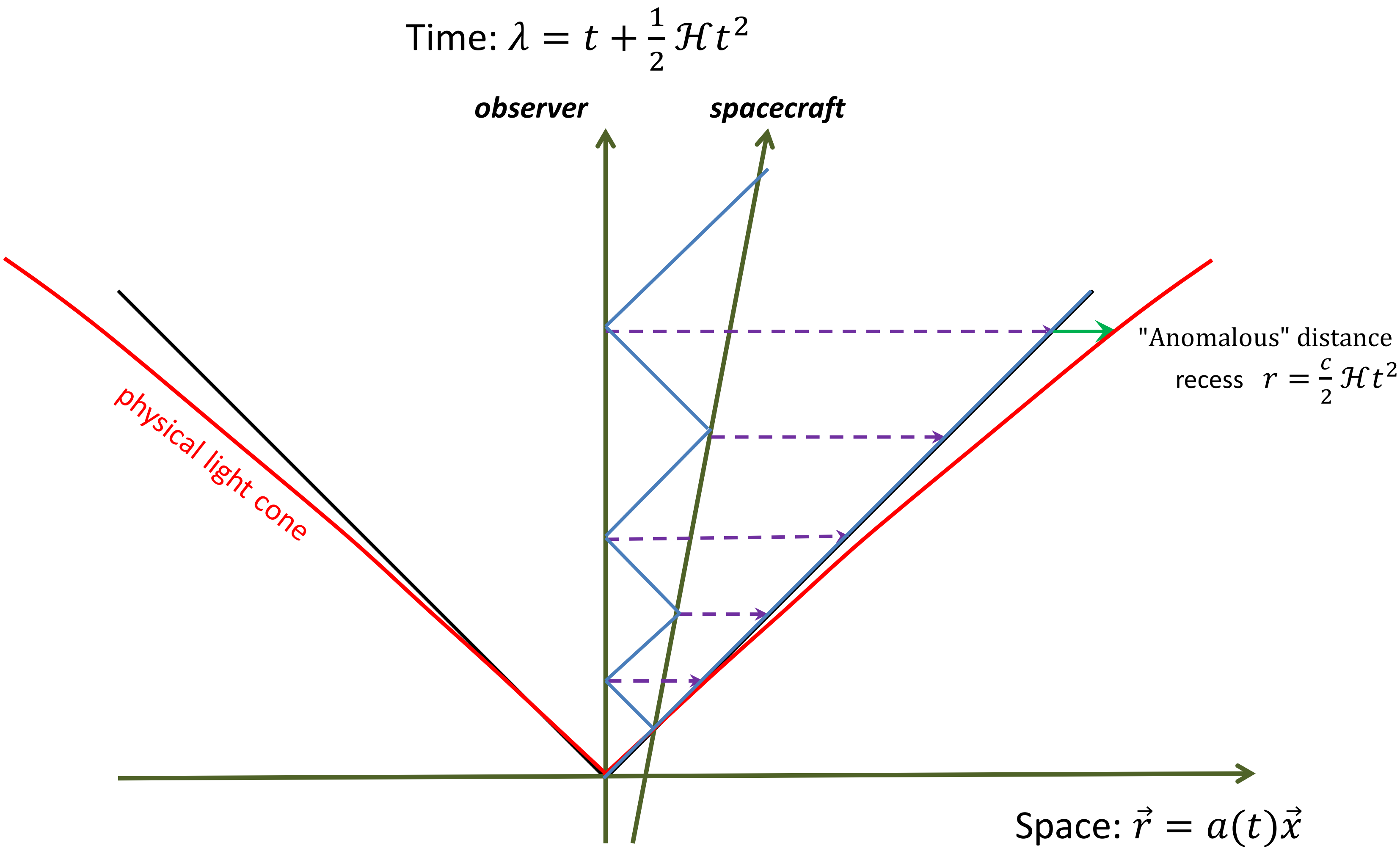
$$\frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{ref}}} = 1 - \frac{2}{c}\int_0^t [a_r(t') - \mathcal{H}c]dt' \Longrightarrow \frac{\nu_{\mathrm{obs}} - \nu_{\mathrm{mod}}}{\nu_{\mathrm{ref}}} = 2\mathcal{H}t$$

blue shift

$$a_{\mathrm{spacecraft}} = a_{\mathrm{model}} - \mathcal{H}c$$

The constant "anomalous acceleration"
$a_{anomaly} = -\mathcal{H}c$ ***is directed toward observer.***
Model ephemeris is fully based on ranging.

Minkowski diagram of the integrated Doppler tracking



Hubble expansion in the solar system?

- **Range measurements**

$$c(t_2 - t_1) = |\vec{r}_2 - \vec{r}_1| - \frac{H}{2c}(r_2^2 - r_1^2)$$

The Hubble correction is $10^{-7}\ cm$ for LLR and 0.1 *cm* for Mars-Earth ranging. Ranging agrees with the new definition of au.

- **Spacecraft Doppler tracking**

$$\frac{\nu_{\mathrm{obs}} - \nu_{\mathrm{model}}}{\nu_0} = 2Ht$$

- **It is plausible that this equation explains the origin of the "anomalous acceleartion" $a_P = -Hc$ visible as a tiny blue Doppler shift of radio frequency on the top of a red shift due to the (outward) radial velocity of the Pioneer spacecraft.**

Prospects

- There were many attempts to find out the “spacetime force” acting on Pioneer spacecraft but it was never found. The “spacetime force” does exist. It is exerted on photons in the physical coordinates $(t, \vec{r})$.

- It is feasible that the integrated effect of this cosmological “inertial force” may be observed and tested in a Doppler tracking experiment designed for a dedicated space mission.

- If the result of this lecture is validated, we get a direct experimental access to the Hubble constant besides cosmological observations.

THANK YOU!

More details:

Archive : http://arxiv.org/abs/1207.3873

Physical Review D: http://prd.aps.org/abstract/PRD/v86/i6/e064004