Light as a probe of fundamental physics in space

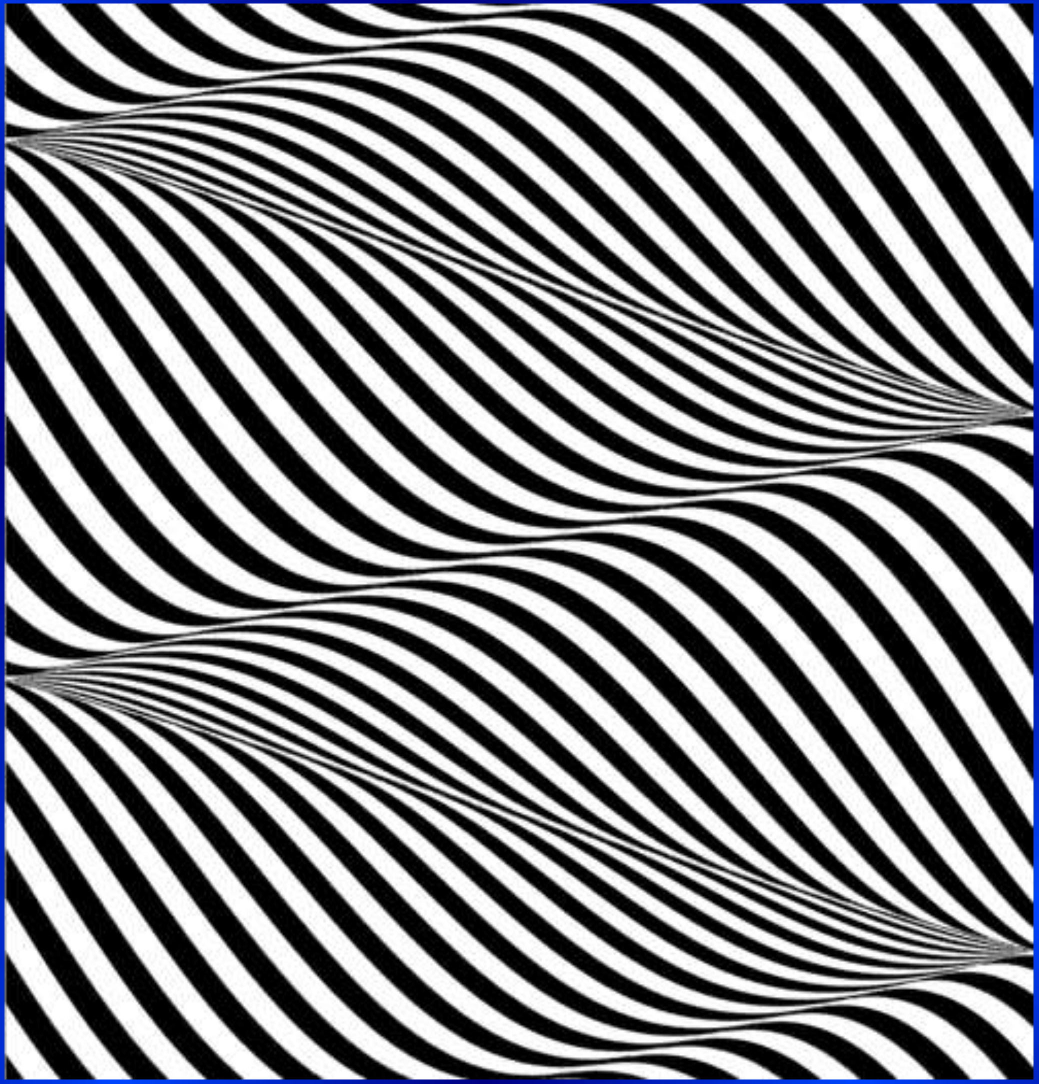
Angelo Tartaglia
Politecnico di Torino and INFN

Outline of the talk

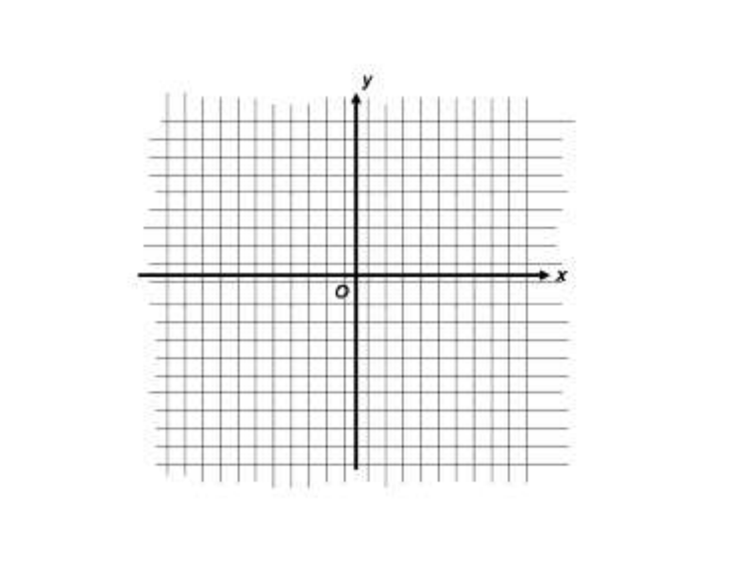
- Space-time and geodesics
- Null geodesics
- Space-time geodesy
- Light as a probe of the local structure of space-time
- Experiments in space

Space-time and geodesics

Families of geodesics map any regular space- time patch

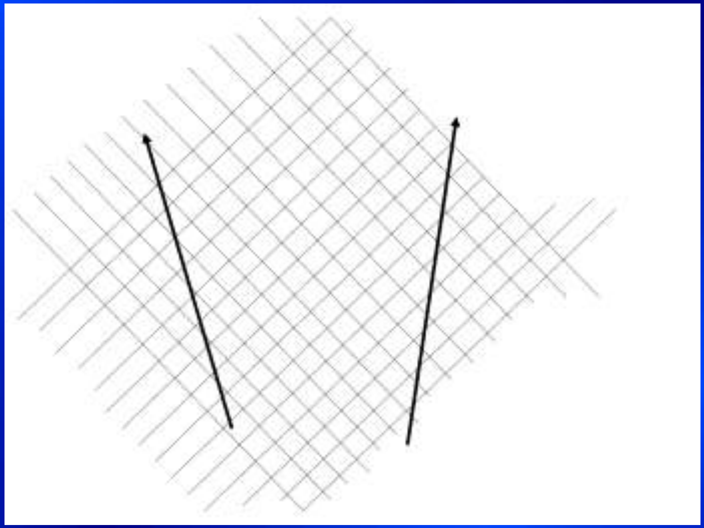


Null or light coordinates



Cartesian grid

Light rays grid



Wave vectors

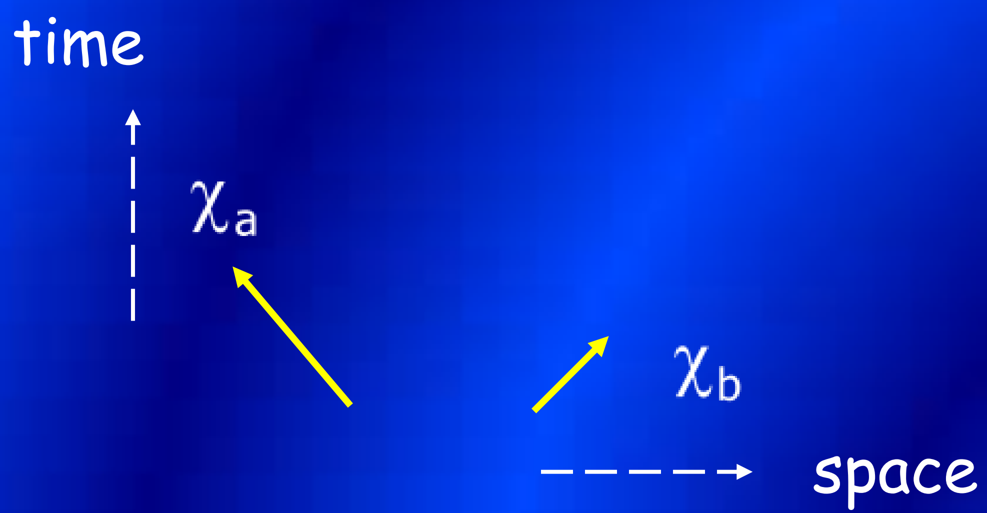
A null geodesic has a null tangent (wave) vector:

$$\chi = cT\left(1, \cos\alpha, \cos\beta, \cos\gamma\right) = cT\left(1, \hat{n}\right)$$

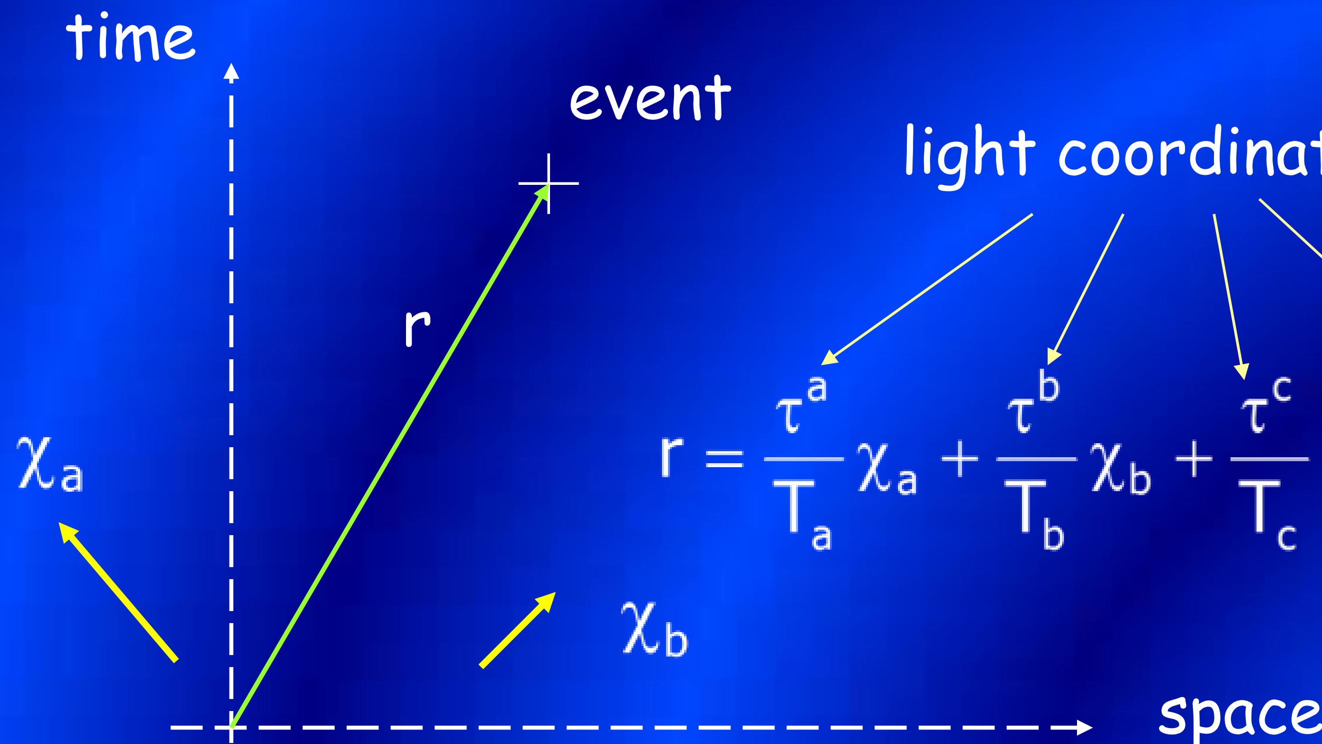
$$\chi^2 = 0$$

Null bases

$\chi_a, \quad \chi_b, \quad \chi_c, \quad \chi_d$



Positioning in space-time



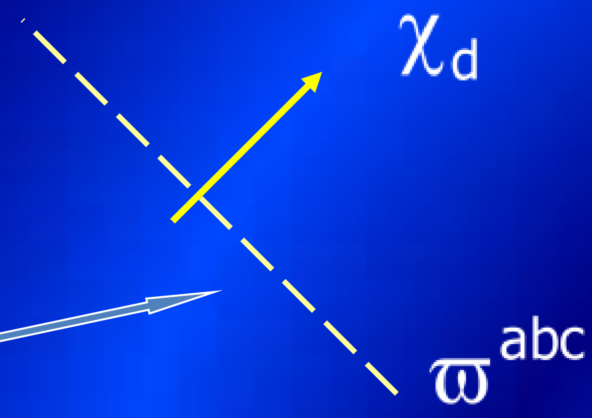
$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

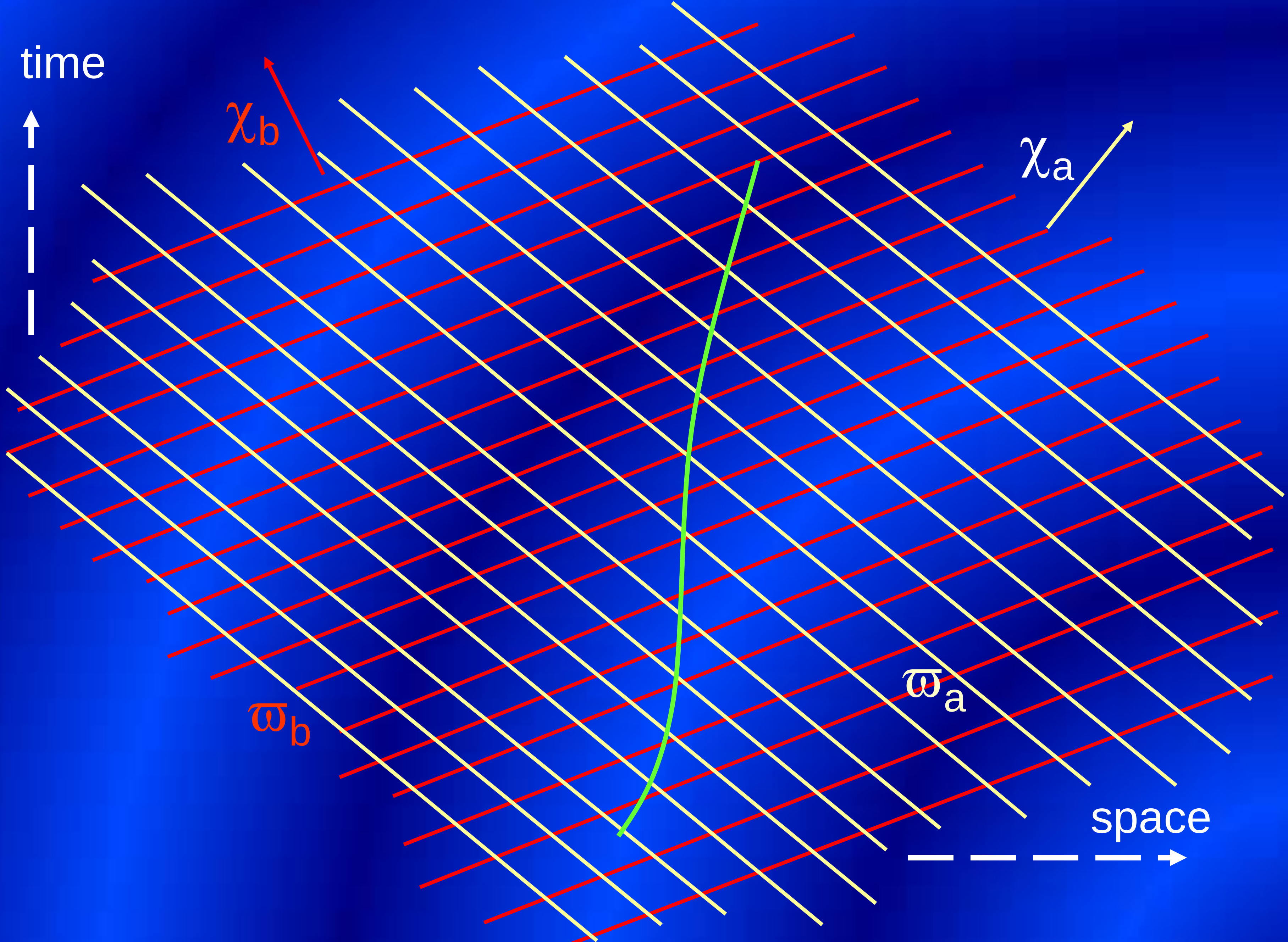
Null wave fronts

Null hypersurface

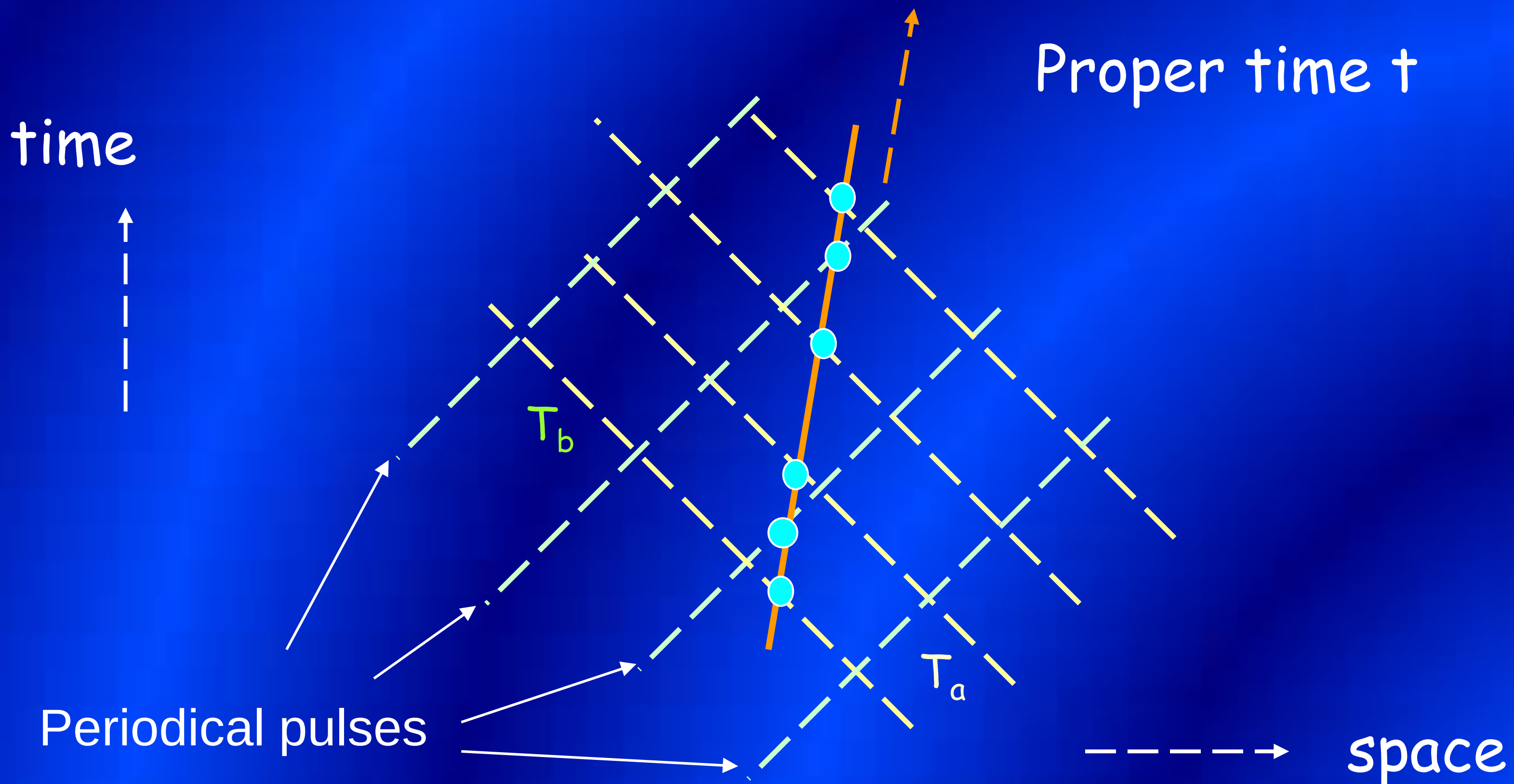
$$\varpi^{abc} = \varepsilon^{abcd}\chi_d$$

Locally a three-dimensional hyperplane




time
χ_b
χ_a
ϖ_a
ϖ_b
space

Locally uniform motion



Light coordinates of an event

$$\tau_{a,b,c,d} = \left[(n + x)T\right]_{a,b,c,d}$$

integer

From simple linear equations

Uncertainty depends on clock

$$\left|\delta x\right| \leq 4\left(\frac{1}{t_{i,i+4n}} + \frac{t_{i,i+1}}{t^2_{i,i+4n}}\right)\delta t$$

As big as allowed by the linearity of the worldline

Accelerated motion

$$x^a = \frac{u^a}{T^a} t + \frac{1}{2} \frac{a^a}{T^a} t^2 + \ldots$$

Four-velocity

Four-acceleration

Maximum integration time $\quad t_{max} = \sqrt{2 \left| \frac{u^a}{a^a} \right| \delta t}$

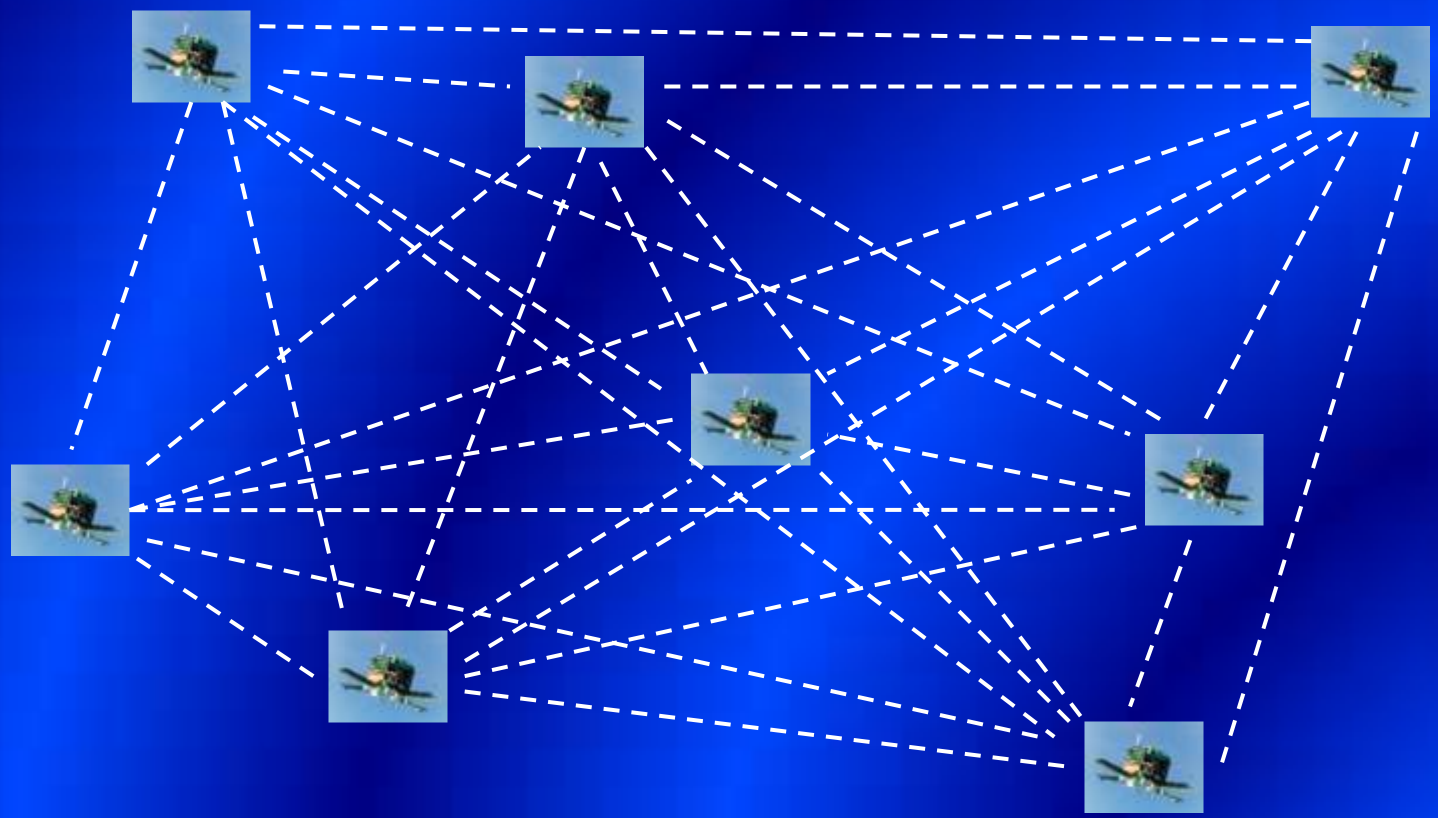
A gravitational field

The gravitational field shows up when:

$$\left| u \cdot \nabla \Phi \right| \geq 4 \frac{\delta t}{t^2}$$

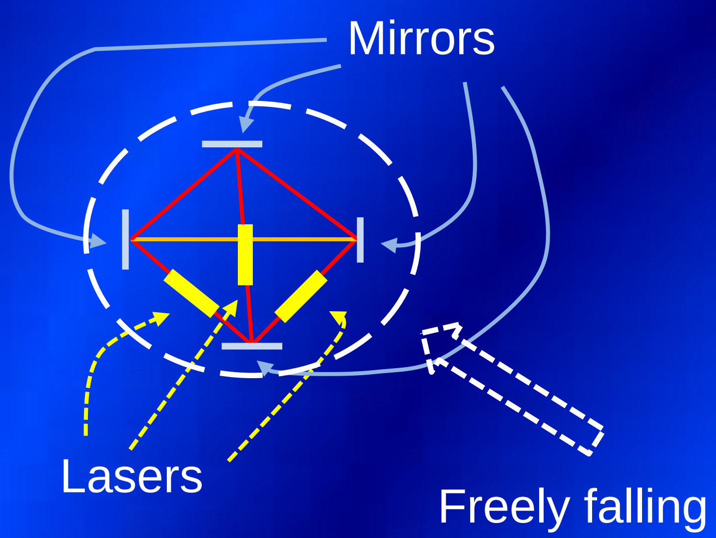
Gravitational potential

Self positioning of a swarm of satellites (space-time geodesy)



Space-time geodesy

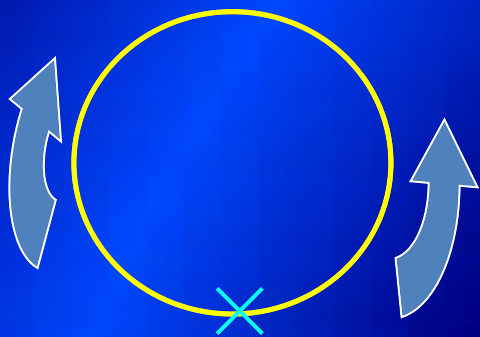
Locally probing the curvature



Tetrahedron with four triangular ringlasers

The effect on frequency

Closed loop (shape irrelevant)



Counterrotating light beams

Time of flight difference

$$\delta T = T_+ - T_- = -2\oint \frac{g_{0\phi}}{g_{00}}\, d\phi \neq 0$$

$$\delta\tau_0 = -2\sqrt{g_{00}}\oint \frac{g_{0\phi}}{g_{00}}\, d\phi \neq 0$$

$$\Delta f = 4\frac{S}{\lambda P}\left[\frac{\omega}{c}\left(1+\frac{3}{2}\frac{GM}{c^2R}-\frac{3}{4}\sqrt{\frac{GM}{c^2R}}+\frac{5}{2}\frac{\omega^2R^2}{c^2}\right)\hat{n}_a\cdot\hat{n}_S-\frac{1}{2}\frac{(GM)^{3/2}}{c^3R^{5/2}}\hat{n}_\theta\cdot\hat{n}_S-\left(\frac{GJ}{c^3R^3}\right)\hat{n}_\vartheta\cdot\hat{n}_S\right]$$

Beat frequency (equatorial orbit)

Corotating device

$$\omega = \Omega = \sqrt{G\frac{M}{R^3}}$$

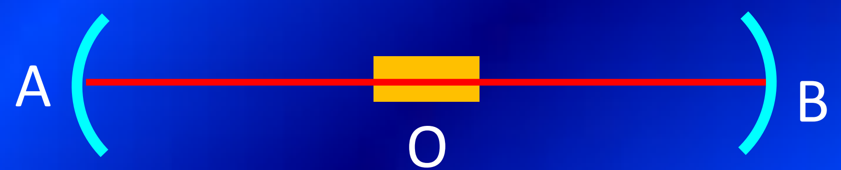
$$\Delta f = 4\frac{S}{\lambda P R}\sqrt{\frac{GM}{R}}\left[\left(1-\frac{3}{4}\sqrt{\frac{GM}{c^2R}}+4\frac{GM}{c^2R}\right)\hat{n}_a\cdot\hat{n}_S - \frac{1}{2}\frac{GM}{c^2R}\hat{n}_\theta\cdot\hat{n}_S - \chi\left(\frac{\sqrt{GMR}\,\Omega_\oplus}{c^2}\right)\hat{n}_\vartheta\cdot\hat{n}_S\right]$$

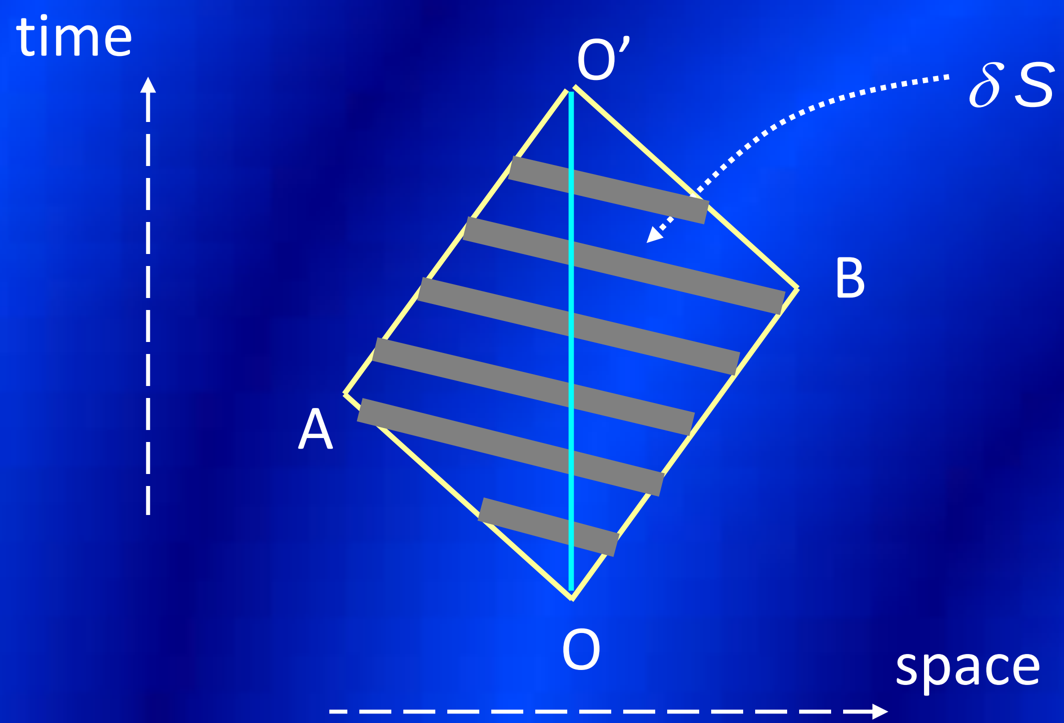
$\sim 10^{-4}$

$\sim 10^{-9}$

$\sim 10^{-12}$

A linear cavity





The effect of curvature and anisotropy

$$\delta F^{\mu\nu} = \left(R^{\mu}{}_{\varepsilon 0i} F^{\varepsilon\nu} + R^{\nu}{}_{\varepsilon 0i} F^{\mu\varepsilon}\right) \delta S^{0i}$$

Electromagnetic tensor

Riemann tensor

Space-time area
spanned by the cavity

Depends on the orientation of the cavity

Linearized Riemann tensor. An example:

$R^{\phi}{}_{r0\theta}$

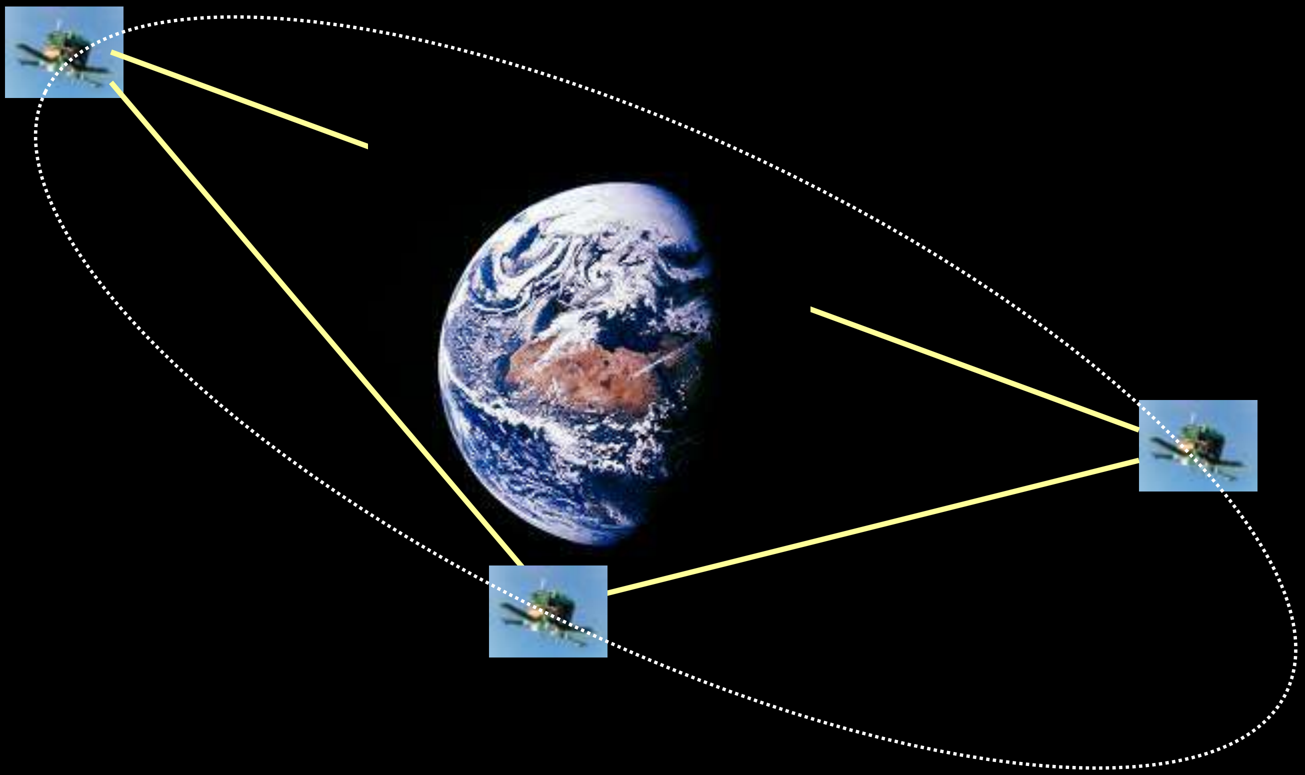
Length of the cavity

$$\delta F^{\vartheta\phi} \cong \left[\left(\frac{(GM)^{3/2}}{c^3 R^{7/2}} - 3\frac{GJ}{c^3 R^4}\right)\frac{\cos\vartheta}{\sin^2\vartheta}\right]\frac{l^2}{R}F^{\vartheta r}$$

Change in the radial component of the magnetic field

East-West component of the magnetic field

Orbital gyro



Arrival times difference

Equatorial orbit

$$\Delta t_0 \cong -6\frac{R}{c}\left[\left(24\sqrt{\frac{2}{\sqrt{3}}}+3\sqrt{\frac{\sqrt{3}}{2}}\right)\left(\frac{GM}{c^2R}\right)^{\frac{3}{2}}-2\frac{\pi}{3}\frac{GM}{c^2R}+4\frac{GJ}{c^3R^2}+\sqrt{2\sqrt{3}}\sqrt{\left(\frac{GM}{c^2R}\right)}\right]$$

Pulses

Beat frequency

Equatorial orbit

$$\Delta f = \frac{c^2}{\lambda P} \Delta t_0 = \frac{c^2}{3\sqrt{3}\lambda R} \Delta t_0$$

Length of the loop

PPN parameters

$$ds^2 = \left(1 - 2\frac{GM}{c^2 r}\right) d\tau^2 - \left(1 + 2\gamma\frac{GM}{c^2 r}\right) dr^2 - r^2 d\vartheta^2$$

$$- r^2 \sin^2\vartheta \, d\phi^2 + \left(4 + 4\gamma + \alpha_1\right)\frac{GJ}{2c^3 r} \sin^2\vartheta \, d\tau d\phi$$

$$2M \rightarrow (1+\gamma)M \qquad 2J \rightarrow \left(1 + \gamma + \frac{\alpha_1}{4}\right) J$$

Conclusion

- Light is an intrinsically relativistic probe of space-time
- Closed paths of light in space (and space-time) evidence general relativistic effects either by
  - Interferometric techniques
    or
  - Beat frequency measurements
- Electromagnetic pulses plus time of arrival times measurements outline a relativistic autonomous space navigation system.